

General one-loop renormalization group evolutions and electroweak symmetry breaking in the $(M+1)$ SSM

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Abstract

We study analytically the general features of electroweak symmetry breaking in the context of the Minimal Supersymmetric Standard Model extended by one Higgs singlet. The exact analytical forms of the renormalization group evolutions of the Yukawa couplings and of the soft supersymmetry breaking parameters are derived to one-loop order. They allow on one hand controllable approximations in closed analytical form, and on the other a precise study of the behaviour of infrared quasi fixed point regimes which we carry out. Some of these regimes are shown to be phenomenologically inconsistent, leading to too small an effective μ -parameter. The remaining ones serve as a suitable benchmark to understand analytically some salient aspects, often noticed numerically in the literature, in relation to the electroweak symmetry breaking in this model. The study does not need any specific assumption on $\tan \beta$ or on boundary conditions for the soft supersymmetry breaking parameters, thus allowing a general insight into the sensitivity of the low energy physics to high energy assumptions.

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1 Introduction.

Extensions of the Higgs sector of the standard model or of the minimal supersymmetric standard model (MSSM) [1], is a suitable framework to assess the phenomenology of the search for Higgs-like particles, and in a wider context, that of the supersymmetric partners of these particles. The next to MSSM, dubbed hereafter (M+1)SSM [2, 3, 4, 5, 6], where one $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge singlet supermultiplet is added [7] to the MSSM, has attracted interest, initially as a framework for a natural solution to the so-called μ -problem [2] and later on as a source for interesting phenomenology which can differ from that of the MSSM. Due to the modification of the Higgs sector, the phenomenology and the upper bound on the mass of the lightest (observable) Higgs is altered [8], while the modified neutralino sector can lead to unconventional signatures for the sparticle searches [9].

Our main concern in this paper is the pattern of electroweak symmetry breaking (EWSB) and the dynamical generation of the μ parameter. As was initially noted in [10] and studied intensively [10, 11], the v.e.v. of the gauge singlet field tends to be much larger than those of the two Higgs doublets, implying the tendency for the singlet chiral superfield to decouple from the other superfields of the theory. This means that, apart from some ranges of the parameter space where the singlino is the lightest supersymmetric particle, the features of the MSSM are basically shared by its minimal extension. However, the analysis of the issues in (M+1)SSM was mostly done numerically, or, when analytically, only for small $\tan\beta$ [11] (no Yukawa couplings for the tau and bottom were considered). Furthermore this is often done in the framework of universality of the soft supersymmetry (susy) breaking parameters, or in some scenarios with mild non-universality which gave results comparable to the ones with universality [12].

In view of the importance of such features (and possibly their generalization to an extended singlet sector), it is important to attempt an understanding of the generic pattern for the dynamical determination of the v.e.v. of the singlet, without any specific assumptions about the GUT-scale boundary conditions or the magnitude of $\tan\beta$. In this paper we will address this issue fully analytically and from two complementary sides: firstly the analytical evolution of the various Yukawa (and gauge) couplings and the soft susy breaking parameters, as well as the determination of the regimes of least sensitivity to initial conditions, *i.e.* in the vicinity of the infrared quasi fixed points³(IRQFP); secondly the study of the EWSB equations in compelling regimes and the interplay between the magnitude of the dynamically determined Higgs doublet mixing parameter $\hat{\mu}$, the experimental lower bounds on chargino masses, and the relaxation of universality of the soft parameters.

The paper is organized as follows: In section 2 we recall the basic ingredients of the (M+1)SSM and introduce our notations. In section 3 we deal with the renormalization group equations (RGEs), give the analytical integrated forms of their solutions along lines similar to [14, 15] and classify the ensuing IRQFPs regimes. Four regimes are found

³ another possibility of least sensitivity, not considered here, could be the occurrence of focus points at phenomenologically acceptable energy scales much like in the MSSM[13].

generalizing the MSSM case [16]. Some numerical illustrations of the IRQFPs regimes are given in section 4. Section 5 is devoted to the study of EWSB constraints and we conclude in section 6. More technical derivations and results are given in Appendices.

From a different standpoint, it is worth keeping in mind potential difficulties that can arise in the (M+1)SSM in relation to the appearance of cosmologically problematic domain-wall solutions. We, however, bypass in this paper such problems and possible solutions to them [17]. Another interesting feature of the (M+1)SSM on which we will not dwell is the possibility to break spontaneously CP symmetry. In this paper we will assume, without further reference, to be in regions of parameter space where CP is broken neither explicitly nor spontaneously [18].

2 The (M+1)SSM.

In this model, the Higgs sector is constituted by two Higgs doublets H_1 and H_2 , and one singlet S . The spectrum, compared to MSSM is richer (one more CP even, CP odd Higgs field, and one more neutralino). Introducing as in the MSSM the matter fields (Q, T, B, E, L) the superpotential reads

$$W = \lambda \hat{S} \hat{H}_1 \cdot \hat{H}_2 + \frac{\kappa}{3} \hat{S}^3 + y_t \hat{T} \hat{Q} \cdot \hat{H}_2 + y_b \hat{B} \hat{Q} \cdot \hat{H}_1 + y_\tau \hat{E} \hat{L} \cdot \hat{H}_1 \quad (2.1)$$

where the dot product represents the SU(2) scalar product, and the superfields \hat{T} (\hat{B}) respectively, the left handed antitop(antibottom), \hat{E} , the left-handed antitau, and \hat{Q} (\hat{L}) the left handed doublets for quarks(leptons). All the parameters in the superpotential are dimensionless, and a mass term is forbidden by a discrete Z_3 symmetry. This symmetry also prevents S to take large v.e.v. ($\langle S \rangle$) and an effective μ parameter is generated ($\hat{\mu} = \lambda \langle S \rangle$) of the order of magnitude of 100 GeV. But, when S develops a v.e.v. this discrete symmetry is broken, and domain wall solutions appear. It is well known that such topological defects are excluded by cosmology. Possible solutions to this are given in [17].

Supersymmetry breaking is parameterized by the so-called soft SUSY terms involving trilinear couplings (A 's), scalar and gaugino masses (m 's and M 's). In (2.1) \hat{H}_1, \hat{H}_2 etc. represent the superfields and now, although H_1, H_2 etc represent its scalar component, λ_1, λ_2 and λ_3 are the $U(1)_Y, SU(2)_L$ and $SU(3)_c$ gauginos respectively.

$$\begin{aligned} \mathcal{L}_{\text{soft}} = & M_1 \lambda_1 \lambda_1 + M_2 \lambda_2 \lambda_2 + M_3 \lambda_3 \lambda_3 + (\lambda A_\lambda S H_1 \cdot H_2 + \frac{\kappa}{3} A_\kappa S^3 \\ & + y_t A_t T Q \cdot H_2 + y_b A_b B Q \cdot H_1 + y_\tau A_\tau E L \cdot H_1 + \text{h.c.}) \\ & + m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_S^2 |S|^2 + m_Q^2 |Q|^2 \\ & + m_T^2 |T|^2 + m_B^2 |B|^2 + m_E^2 |E|^2 + m_L^2 |L|^2 \end{aligned} \quad (2.2)$$

Finally, let us write for later reference, the tree-level scalar potential in the neutral

Higgs sector including the F-terms from Eq.(2.1), the D-terms and the contributions from the soft terms Eq.(2.2),

$$\begin{aligned}
V = & m_1^2 |H_1^0|^2 + m_2^2 |H_2^0|^2 + m_s^2 |S|^2 + \frac{\bar{g}^2}{4} (|H_1^0|^2 - |H_2^0|^2)^2 \\
& + |\kappa S^2 + \lambda H_1^0 H_2^0|^2 + \lambda^2 |S|^2 (|H_1^0|^2 + |H_2^0|^2) \\
& + (A_\lambda \lambda S H_1^0 H_2^0 + A_\kappa \frac{\kappa}{3} S^3 + \text{h.c.})
\end{aligned} \tag{2.3}$$

where $\bar{g} \equiv \sqrt{(g_1^2 + g_2^2)}/2$, g_1, g_2 being the gauge couplings associated respectively to $U(1)_Y, SU(2)_L$ and we consider only real valued couplings and mass parameters.

3 Analytical solution for the renormalization group equation.

3.1 Exact evolutions to one-loop order

Using the notations $\alpha_i = \frac{g_i^2}{16\pi^2}$, $i = 1, 2, 3$; $Y_j = \frac{y_j^2}{16\pi^2}$, $j = t, b, \tau$, and $Y_{(\lambda, \kappa)} = \frac{(\lambda^2, \kappa^2)}{16\pi^2}$, where g_1, g_2 and g_3 denote respectively the $U(1)_Y, SU(2)_L$ and $SU(3)_c$ gauge coupling constants, one can write down the one-loop RG equations as [3, 21]

$$\begin{aligned}
\dot{\alpha}_i &= -b_i \alpha_i^2, \\
\dot{M}_i &= -b_i \alpha_i M_i, \\
\dot{Y}_k &= Y_k \left(\sum_i c_{ki} \alpha_i - \sum_l a_{kl} Y_l \right) \\
\dot{A}_k &= - \left(\sum_i c_{ki} \alpha_i M_i + \sum_i a_{ki} Y_i A_i \right) \\
\dot{\Sigma}_k &= 2 \sum_i c_{ki} \alpha_i M_i \bar{M}_i - \sum_i a_{ki} Y_i (\Sigma_i + A_i \bar{A}_i)
\end{aligned} \tag{3.1}$$

where $k = t, b, \tau, \lambda, \kappa$, $\dot{} \equiv d/dt$, $t = \log M_{GUT}^2/Q^2$, the numerical coefficients a 's, b 's and c 's are given in Appendix A, and the Σ_k are defined in Eq.(3.11).

Though the RGEs for the Yukawa couplings Eqs.(3.1) do not have explicit analytic solutions, they can be solved iteratively as it has been demonstrated in [14] through the use of some auxiliary functions.

Together with the gauge couplings, the general solutions for the Yukawa couplings read [14]

$$\begin{aligned}
\alpha_i &= \frac{\alpha_i^0}{1 + b_i \alpha_i^0 t} \\
Y_k &= \frac{Y_k^0 u_k}{1 + a_{kk} Y_k^0 \int_0^t u_k}
\end{aligned} \tag{3.2}$$

where the auxiliary functions u_k are given by

$$u_k(t) = \frac{E_k(t)}{\prod_{j \neq k} (1 + a_{jj} y_j^0 \int_0^t u_j)^{a_{kj}/a_{jj}}} \quad (3.3)$$

and

$$E_k(t) = \prod_{i=1}^3 (1 + b_i \alpha_i^0 t)^{\frac{c_{ki}}{b_i}} \quad (3.4)$$

Specifying to the model under consideration one finds (see Appendix A for tabulation of the coefficients)

$$u_t = \frac{E_t}{(1 + 6Y_b^0 \int u_b)^{\frac{1}{6}} (1 + 4Y_\lambda^0 \int u_\lambda)^{\frac{1}{4}}} \quad (3.5)$$

$$u_b = \frac{E_b}{(1 + 6Y_t^0 \int u_t)^{\frac{1}{6}} (1 + 4Y_\tau^0 \int u_\tau)^{\frac{1}{4}} (1 + 4Y_\lambda^0 \int u_\lambda)^{\frac{1}{4}}} \quad (3.6)$$

$$u_\tau = \frac{E_\tau}{(1 + 6Y_b^0 \int u_b)^{\frac{1}{2}} (1 + 4Y_\lambda^0 \int u_\lambda)^{\frac{1}{4}}} \quad (3.7)$$

$$u_\lambda = \frac{E_\lambda}{(1 + 6Y_t^0 \int u_t)^{\frac{1}{2}} (1 + 6Y_b^0 \int u_b)^{\frac{1}{2}} (1 + 4Y_\tau^0 \int u_\tau)^{\frac{1}{4}} (1 + 6Y_\kappa^0 \int u_\kappa)^{\frac{2}{3}}} \quad (3.8)$$

$$u_\kappa = \frac{E_\kappa}{(1 + 4Y_\lambda^0 \int u_\lambda)^{\frac{3}{2}}} \quad (3.9)$$

We also use the shorthand notation \int to mean \int_0^t and drop out for simplicity any explicit reference to the scale t in all running quantities. Let us stress that (3.2) give the exact solution to Y_k and α_i while the u_k 's given in (3.5)-(3.9), although solved formally in terms of the E_k 's and Y_k^0 's as continued integrated fractions, should, in practice, be solved iteratively. Yet, the important gain here is threefold: (i) as shown in [14], the convergence of the successive iterations to the exact solution can be fully controlled analytically in terms of the initial values of the Yukawa couplings. This means that one can in practice use truncated iterations of Eqs.(3.5)-(3.9), say to first order, and thus obtain very good analytical approximations to the exact solutions (ii) the structure of the solutions is explicit enough to allow a thorough study of some limiting regimes as we will see in the next section (iii) furthermore, all these nice features will be naturally passed to the solutions for the soft SUSY breaking parameters since the latter will be obtained from (3.2) through the method of [15].

To obtain the solutions for the soft parameters one starts from those of the couplings of the supersymmetric rigid theory, and makes the substitutions

$$\begin{aligned} \alpha &\rightarrow \alpha_i (1 + M_i \eta + \bar{M}_i \bar{\eta} + 2M_i \bar{M}_i \eta \bar{\eta}), \\ Y_k &\rightarrow Y_k (1 - A_k \eta - \bar{A}_k \bar{\eta} + (\Sigma_k + A_k \bar{A}_k) \eta \bar{\eta}), \end{aligned} \quad (3.10)$$

where the M_i 's are the gaugino masses, the A_k 's the scalar soft trilinear coupling constants and Σ_k are the following combinations of the soft masses

$$\begin{aligned}\Sigma_t &= m_Q^2 + m_T^2 + m_2^2, \quad \Sigma_b = m_Q^2 + m_B^2 + m_1^2, \quad \Sigma_\tau = m_L^2 + m_E^2 + m_1^2, \\ \Sigma_\lambda &= m_1^2 + m_2^2 + m_S^2, \quad \Sigma_\kappa = 3m_S^2\end{aligned}\tag{3.11}$$

and $\eta = \theta^2$ and $\bar{\eta} = \bar{\theta}^2$ are the spurion fields depending on the Grassmannian parameters $\theta_\alpha, \bar{\theta}_{\dot{\alpha}}$ ($\alpha = 1, 2$).

Performing the substitution (3.10) in (3.2) and identifying the coefficients of the resulting polynomial in η and $\bar{\eta}$, the linear term in η gives us the solution for M_i and A_k and the $\eta\bar{\eta}$ terms the ones for Σ_k . (For simplicity, we do not consider here CP-violating effects and take all the soft parameters to be real valued.) The resulting exact solutions look similar to those for the rigid couplings

$$\begin{aligned}M_i &= \frac{M_i^0}{1 + b_i \alpha_i^0 t}, \\ A_k &= -e_k + \frac{A_k^0/Y_k^0 + a_{kk} \int u_k e_k}{1/Y_k^0 + a_{kk} \int u_k}, \\ \Sigma_k &= \xi_k + A_k^2 + 2e_k A_k - \frac{(A_k^0)^2/Y_k^0 - \Sigma_k^0/Y_k^0 + a_{kk} \int u_k \xi_k}{1/Y_k^0 + a_{kk} \int u_k},\end{aligned}\tag{3.12}$$

where the new auxiliary functions e_k and ξ_k have been introduced and are given in Appendix B, Eqs.(B.1) - (B.5).

In the particular case where $Y_b = Y_\tau = Y_\lambda = Y_\kappa = 0$ Eqs.(3.2)–(3.12) reduce to the exact well known solutions in the “small $\tan\beta$ ” regime.

3.2 Large Yukawa regimes

In this section, we study various regimes of large Yukawa couplings, and their incidence on the solutions written above. In [16], iterated and analytical solutions to the RGE of the MSSM allowed an extensive study in the Infra-Red-Quasi-Fixed Point (IRQFP) [19] regime, i.e., a regime where $Y_{i=t,b,\tau}^0$ go to infinity. In the case of the (M+1)SSM, we want to see where and how strong could be the influence of the singlet on the MSSM solutions. For that, we have studied 4 different regimes including the two new Yukawa couplings Y_λ^0 and Y_κ^0 , always considering $Y_{i=t,b,\tau}^0 \rightarrow \infty$ in order to compare directly with the MSSM case [16]:

- regime 1 : Y_λ^0 and Y_κ^0 are finite
- regime 2 : Y_λ^0 is finite and Y_κ^0 goes to infinity
- regime 3 : Y_λ^0 goes to infinity and Y_κ^0 is finite

- regime 4 : Y_λ^0 and Y_κ^0 go to infinity

Let us make here some comments about the meaning of “infinite” initial conditions. Formally this means that we reach a Landau pole, and that the corresponding low energy values are at the edge of the triviality bounds. This clearly implies that perturbativity breaks down somewhere between the low and high (presumably GUT) scales. In practice though, we have checked numerically (see section 4) that we reach the effective fixed point (EFP) behaviour very quickly. A value of Y_i^0 of 0.1 is already in the EFP regime. So, there is no problem concerning the perturbativity at high scale. (In terms of the Yukawa couplings of the Lagrangian Eqs.(2.1) – (2.3) we never take initial values larger than 5 which corresponds to $Y's \lesssim 0.16$).

When some of the Yukawa couplings become large, one is tempted to drop altogether the 1's in the corresponding $(1 + Y_i^0 \int u_i)$'s appearing in (3.2) and (3.5)-(3.9) and to expect a typical limiting behaviour for the u 's of the form⁴

$$u_i^\infty \sim \frac{1}{(Y^0)^{p_i}} \quad (3.13)$$

where Y^0 is a large Yukawa coupling.

However, the situation is not so simple because of the implicit dependence of each u_i on the full set of u 's in a continued fraction like way, especially when not all the Yukawa couplings become simultaneously large, as is the case in some of the regimes we consider here. Although it is easy to understand intuitively the validity of (3.13) if a similar form is obtained at some n^{th} order iteration of the truncated approximation to Eqs.(3.5)-(3.9), *and provided that the p_i 's verify $0 < p_i < 1$ order by order*, a more careful study is required to control the magnitudes of these powers.

Indeed, in contrast to the MSSM case [16], some of the p_i 's can be larger than one due to the singlet sector Eqs.(3.8), (3.9). Technically, one has to solve (C.4) of Appendix C to which the reader is referred for more technical discussions and details of the derivation.

Here we give directly the final results for the running Yukawa couplings in the various IRQFP regimes:

$$Y_{i=t,b,\tau}^{\text{FP}} = \frac{u_i^{\text{FP}}}{a_{ii} \int u_{t,b,\tau}^{\text{FP}}} \text{ regimes 1, 2, 3 and 4} \quad (3.14)$$

$$Y_\lambda^{\text{FP}} = \begin{cases} \frac{Y_\lambda^0 u_\lambda^{\text{FP}}}{(Y_t^0)^{p_\lambda}} \sim 0 & \text{regimes 1, 2 and 4} \\ \frac{u_\lambda^{\text{FP}}}{4 \int u_\lambda^{\text{FP}}} & \text{regime 3} \end{cases} \quad (3.15)$$

⁴note also that this approximation does not work at scales too close to the initial (GUT) scale because of the term $\int_0^t u_i$, but it does hold for a typical electroweak scale $t \sim 66$.

$$Y_\kappa^{\text{FP}} = \begin{cases} \frac{Y_\kappa^0}{1+6Y_\kappa^0 t} & \text{regime 1} \\ \frac{1}{6t} & \text{regimes 2 and 4} \\ \frac{Y_\kappa^0 u_\kappa^{\text{FP}}}{(Y_t^0)^{31}} \sim 0 & \text{regime 3} \end{cases} \quad (3.16)$$

where the u_i^{FP} are related to u_i^∞ through

$$u_i^\infty = \frac{u_i^{\text{FP}}}{(Y_t^0)^{p_i}} \quad (3.17)$$

and depend, as well as the p_i 's, on the regime under consideration (see Appendix C).

Let us stress several points here :

(i) the solutions for the $Y_{i=t,b,\tau}^{\text{FP}}$ have the same form as in the MSSM [16]. Nevertheless, the effect of the singlet is implicit in the recursive solutions for the u_i^{FP} Eqs.(C.6) – (C.10).

(ii) $Y_\lambda^{\text{FP}} \sim 0$ in the regimes 1, 2 and 4. This will be important for the electroweak symmetry breaking discussed later on.

(iii) We have an *exact* analytical solution for Y_κ in the regimes 1, 2 and 4, as a function of its initial value at the GUT scale, Y_κ^0 (see the numerical analysis for a more detailed discussion).

(iv) It is also important to emphasize that the u^{FP} Eqs.(C.6) – (C.10) depend only on the ratios of the large initial values of the Yukawa couplings. However, even this dependence drops out completely in Eqs.(3.14) – (3.16), so that the initial conditions are completely screened in the IRQFP regimes as expected. Only a sensitivity to the initial Yukawa couplings that are not large may still occur, like in regime 1 for Y_κ^{FP} , Eq.(3.16).

(v) A comment is in order here about the difference between the IR quasi fixed points we discuss and the exact fixed points studied in [6]. For one thing, the latter exact fixed points, actually exact fixed ratios, exist only in reduced couplings configurations where all gauge couplings and all Yukawa couplings but the top are neglected, while the quasi-fixed points we study are valid without this approximation. Furthermore, the IRQFP's are more likely to influence the evolution from the GUT scale to the electroweak scale than are the exact fixed points [19]. Let us note, however, that in one or the other of our four IRQFP regimes we find either κ or λ to be vanishingly small, Eqs.(3.15), (3.16), similarly to the case of two among the three exact fixed point regimes determined in [6]. Nonetheless, the latter two regimes were found to be infrared repulsive [6], while as one can infer from the structure of the denominator in Eq.(3.2), the top down evolution of the Yukawa couplings tends generically always *towards* the IRQFP's behaviour.

To find the IRQFP behaviour of the soft parameters A_i, Σ_i one can either perform the substitutions Eq.(3.10) in Eqs.(3.14) – (3.16), (C.6) – (C.10), (3.4), or study the large initial Yukawa limit directly from the general solutions Eq.(3.12). Denoting the auxiliary functions e_i of Eqs.(B.1) - (B.5) by e_i^∞ in this limit, we obtain for the A 's

$$A_{i=t,b,\tau}^\infty = -e_i^\infty + \frac{\int u_i^\infty e_i^\infty}{\int u_i^\infty} = A_i^{\text{FP}} \text{ in all regimes} \quad (3.18)$$

$$A_\lambda^\infty = \begin{cases} -e_\lambda^\infty + A_\lambda^0 & \text{regimes 1, 2 and 4} \\ -e_\lambda^{\text{FP}} + \frac{\int u_\lambda^{\text{FP}} e_\lambda^{\text{FP}}}{\int u_\lambda^{\text{FP}}} & \text{regime 3} \end{cases} \quad (3.19)$$

$$A_\kappa^\infty = \begin{cases} -e_\kappa^\infty + \frac{A_\kappa^0 + 6Y_\kappa^0 \int e_\kappa^\infty}{1 + 6Y_\kappa^0 t} & \text{regime 1} \\ -e_\kappa^{\text{FP}} + \frac{\int e_\kappa^{\text{FP}}}{t} & \text{regimes 2 and 4} \\ -e_\kappa^\infty + A_\kappa^0 & \text{regime 3} \end{cases} \quad (3.20)$$

where

$$A_i^{\text{FP}} \equiv -e_i^{\text{FP}} + \frac{\int u_i^{\text{FP}} e_i^{\text{FP}}}{\int u_i^{\text{FP}}} \quad (3.21)$$

and the e_i^∞ are defined in Appendix D Eqs.(D.2) – (D.6). These auxiliary functions depend explicitly on the initial values A_k^0 . In some IRQFP regimes, this dependence cancels out in the running parameters A_i^∞ . Such an independence occurs in all the regimes for $A_{t,b,\tau}$ Eq.(3.18) as was the case in the MSSM [16], in the regime 3 for A_λ Eq.(3.19) and in regimes 2, 3 for A_κ Eq.(3.20). In such cases we have re-expressed the results in terms of new quantities e_i^{FP} which are independent of the initial conditions. The dependence on initial conditions is explicit further in Eqs.(D.8), (D.9).

Similarly, denoting by ξ_i^∞ the auxiliary functions ξ_i Eqs.(B.7) – (B.11) in the large Yukawa limits we obtain for the Σ 's

$$\Sigma_{i=t,b,\tau}^\infty = \Sigma_i^{\text{FP}} \equiv \xi_i^{\text{FP}} + (A_i^{\text{FP}})^2 + 2e_i^{\text{FP}} A_i^{\text{FP}} - \frac{\int u_i^{\text{FP}} \xi_i^{\text{FP}}}{\int u_i^{\text{FP}}} \quad (3.22)$$

$$\Sigma_\lambda^\infty = \begin{cases} \xi_\lambda^\infty + (A_\lambda^\infty)^2 + 2A_\lambda^\infty e_\lambda^\infty + \Sigma_\lambda^0 - (A_\lambda^0)^2 & \text{regime 1, 2 and 4} \\ \xi_\lambda^{\text{FP}} + (A_\lambda^{\text{FP}})^2 + 2e_\lambda^{\text{FP}} A_\lambda^{\text{FP}} - \frac{\int u_\lambda^{\text{FP}} \xi_\lambda^{\text{FP}}}{\int u_\lambda^{\text{FP}}} & \text{regime 3} \end{cases} \quad (3.23)$$

$$\Sigma_\kappa^\infty = \begin{cases} \xi_\kappa^\infty + (A_\kappa^\infty)^2 + 2A_\kappa^\infty e_\kappa^\infty + \xi_\kappa^\infty + \frac{\Sigma_\kappa^0 - (A_\kappa^0)^2 - 6Y_\kappa^0 \int \xi_\kappa^\infty}{1 + 6Y_\kappa^0 t} & \text{regime 1} \\ \xi_\kappa^{\text{FP}} + (A_\kappa^{\text{FP}})^2 + 2e_\kappa^{\text{FP}} A_\kappa^{\text{FP}} - \frac{\int \xi_\kappa^{\text{FP}}}{t} & \text{regimes 2 and 4} \\ \xi_\kappa^\infty + (A_\kappa^\infty)^2 + 2A_\kappa^\infty e_\kappa^\infty + \Sigma_\kappa^0 - (A_\kappa^0)^2 & \text{regime 3} \end{cases} \quad (3.24)$$

where again ξ_i^{FP} have been used instead of ξ_i^∞ for those cases where the dependence on initial conditions is absent in the running Σ 's. The reader is referred to Appendix D for an extended discussion of the relation between $e_i^{\text{FP}}, \xi_i^{\text{FP}}$ and e_i^∞, ξ_i^∞ and for the explicit dependence on initial conditions Eqs.(D.18), (D.19). Some remarks are in order:

(i) A_λ depends on the initial condition A_i^0 in the regimes 1, 2 and 4, but in these regimes $Y_\lambda \sim 0$ Eq.(3.15). Similarly, A_κ depend on the initial values of A_i^0 in the regime 3, but in this regime $Y_\kappa \sim 0$ (3.16). So, we can conclude that, in every regime, the running of the combination $4\pi\sqrt{Y_i}A_i$, $i = \lambda, \kappa$, the one present in the Lagrangian, is indeed screened from the GUT-scale initial conditions.

(ii) We have the *exact* analytical dependence of the soft terms on the initial conditions A_i^0, Σ_i^0 (c.f. Appendix D).

(iii) Whereas the dependence of the ξ^∞ on the products $A_i^0 A_j^0$ and Σ_j^0 is rather complicated, the Σ_i^∞ 's depend only on the Σ_j^0 , except for regime 1 where A_κ^0 is present, see Eqs.(D.18), (D.19). Moreover, this dependence is exactly the same as the dependence of the A_i^∞ on the A_j^0 .

(iv) It is worth noting that the abovementioned sensitivity of $\Sigma_{\lambda,\kappa}^\infty$ to Y_κ^0, A_κ^0 in the regime 1 disappears in all the soft masses, except for the singlet soft mass (m_S), see Eqs.(B.13) –(B.20), (D.36). Moreover, in regimes 1, 2 and 4, where the soft mass of the singlet separates from all the others, Eq.(D.35), the dependence on initial conditions for the soft masses is exactly the same as the one found in [16] in the case of the MSSM. Finally, let us note that it does not seem possible to give the explicit dependence of the A and the Σ 's on gauginos mass initial conditions because the latter come always in scale dependent contributions, Eqs.(B.6), (B.12).

A further point should be made here about the evolution of the soft scalar masses since we address the most general situation beyond universality. In fact, to solve exactly the RGE for the soft masses, we have to consider the complete equations, including a $U(1)$ induced “trace term” S ,

$$(\dot{m}_i^2) = f_i(M_1, M_2, M_3, \Sigma_i, A_i) + T_i \alpha_1 S \quad (3.25)$$

where $f_i(M_{1,2,3}, \Sigma_i, A_i)$ are defined as

$$\begin{aligned} f_t &= \frac{16}{9}\alpha_1 M_1^2 + \frac{16}{3}\alpha_3 M_3^2 - 2Y_t(\Sigma_t + A_t^2) \\ f_b &= \frac{4}{9}\alpha_1 M_1^2 + \frac{16}{3}\alpha_3 M_3^2 - 2Y_b(\Sigma_b + A_b^2) \\ f_Q &= \frac{1}{9}\alpha_1 M_1^2 + 3\alpha_2 M_2^2 + \frac{16}{3}\alpha_3 M_3^2 - Y_t(\Sigma_t + A_t^2) - Y_b(\Sigma_b + A_b^2) \\ f_E &= 4\alpha_1 M_1^2 - 2Y_\tau(\Sigma_\tau + A_\tau^2) \\ f_L &= \alpha_1 M_1^2 + 3\alpha_2 M_2^2 - Y_\tau(\Sigma_\tau + A_\tau^2) \\ f_{H_1} &= \alpha_1 M_1^2 + 3\alpha_2 M_2^2 - Y_\lambda(\Sigma_\lambda + A_\lambda^2) - Y_\tau(\Sigma_\tau + A_\tau^2) - 3Y_b(\Sigma_b + A_b^2) \\ f_{H_2} &= \alpha_1 M_1^2 + 3\alpha_2 M_2^2 - Y_\lambda(\Sigma_\lambda + A_\lambda^2) - 3Y_t(\Sigma_t + A_t^2) \end{aligned}$$

$$f_s = -2Y_\lambda(\Sigma_\lambda + A_\lambda^2) - 2Y_\kappa(\Sigma_\kappa + A_\kappa^2) \quad (3.26)$$

$$S = \sum_{\text{generations}} (m_Q^2 - 2m_U^2 + m_D^2 - m_L^2 + m_E^2) + m_2^2 - m_1^2 \quad (3.27)$$

and

$$T_i = \{1/3, -4/3, 2/3, -1, 2, -1, 1, 0\} \quad (3.28)$$

with $i = \{Q, U, D, L, E, H_1, H_2, S\}$ respectively. From Eqs.(3.25), (3.26), (3.27) and (3.28) one sees that $\dot{S} \propto \alpha_1 S$, so that if S vanishes at some scale t_0 it will vanish identically at any scale t . For instance $S = 0$ at any scale in the case of universality of all soft scalar masses. More generally, S can still be ignored even when universality is relaxed provided that the initial conditions are such that $S(t_0) = 0$. This simplifying configuration was taken up in [16]. Generically, however, one should solve Eq.(3.25) keeping the trace term. This can be easily done by writing the solution for the soft masses as

$$m_i^2 = (m_i^2)_f + (m_i^2)_{Tr} \quad (3.29)$$

where $(m_i^2)_f$ is solution of the equation $(\dot{m}_i^2)_f = f_i(M_1, M_2, M_3, \Sigma_i, A_i)$, and $(\dot{m}_i^2)_{Tr} = T_i \alpha_1 S$. The $(m_i^2)_f$ are then given by Eqs.(B.13), (B.20), and $(m_i^2)_{Tr}$ is just the solution of a linear differential equation system. We obtain

$$(m_i^2)_{Tr} = t_i S_0 ((1 + b_1 \alpha_1^0 t)^{\frac{26}{33}} - 1) \quad (3.30)$$

with

$$t_i = \frac{1}{26} \{1, -4, 2, -3, 6, -3, 3, 0\} \quad (3.31)$$

where $S_0 = S(t = 0)$.

4 Numerical analysis

As we stressed before, one of the advantages of our approach is that we can extract the exact sensitivity to initial conditions in the quasi-fixed point regimes. In this section, we study numerically the solutions for the Yukawa couplings Y_i , the trilinear couplings A_i , and the soft parameters m_i , in the four IRQFP regimes we have identified in section 3. We compare our analytical solutions to the results obtained by a purely numerical resolution of the RGE's, relying on a FORTRAN code which evolves the relevant parameters from the GUT scale to the electroweak scale through an algorithm similar to the one used in [11].

In the case of the Yukawa couplings we will restrain ourselves, for the sake of illustration, to the study of Y_κ . In Fig. 1a we plot Y_κ at the EW scale ($t \simeq 66$) as a function of its value at the GUT scale, for different values of Y_λ^0 and for $Y_{t,b,\tau}^0 = 0.1$. Since Y_κ^0 is varied only in a range of small values, we reach in the same graph regime 1 (finite Y_λ^0 and Y_κ^0) as

well as regime 3 (infinite Y_λ^0 and finite Y_κ^0)⁵. We have also plotted in the same figure the analytical solution (3.16). We thus see, in the small Y_λ^0 region, the numerical agreement with the expected behaviour $Y_\kappa(t=66) \sim \frac{Y_\kappa^0}{1+6Y_\kappa^0 t}$ of regime 1, and $Y_\kappa(66) \sim 0$ for large values of Y_λ^0 in accord with regime 3. Fig. 1b shows the variation of $Y_\kappa(t=66)$ for bigger values of Y_κ^0 (regime 2 and 4). We clearly see that Y_κ tends to the analytical solution $\frac{1}{a_{\kappa\kappa}t}$ (~ 0.0025 for $t=66$) for small values (regime 2) or high values (regime 4) of Y_λ^0 . Evaluating the u_i^{FP} 's up to the third order iteration with a procedure similar to that in [16], we also found very good numerical agreement between (3.14), (3.15) and the FORTRAN code output.

We note finally that our results for Y_κ in regimes 2 and 4 are in perfect agreement with the numerical illustrations of [4], while Y_λ was found to differ slightly. The reason for this numerical difference can be traced back to the fact that Y_κ is completely decoupled from the t, b, τ sector, while Y_λ is closely tied to it. Whence an important influence of the $Y_{t,b,\tau}$ on the running of Y_λ , which modifies significantly its low scale value.

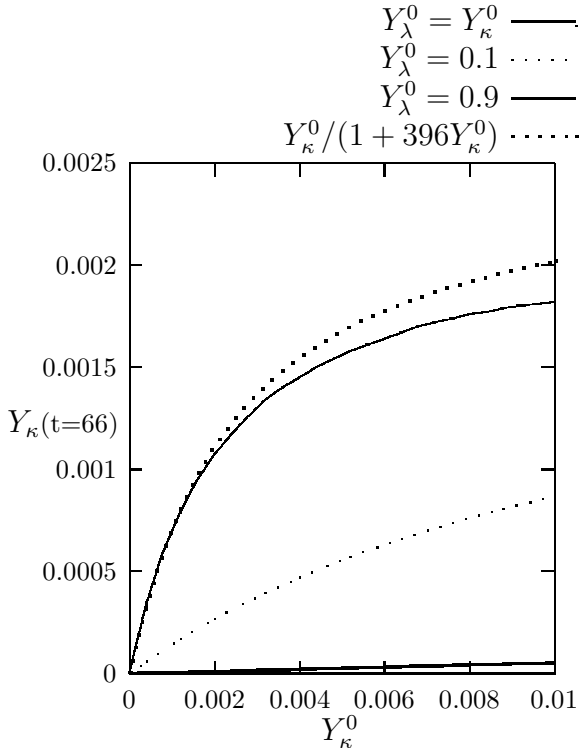


Fig. 1.a

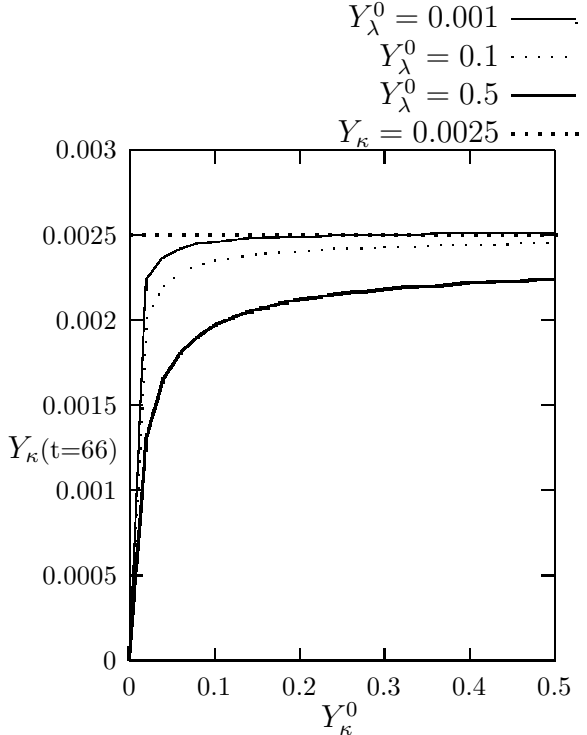


Fig. 1.b

In figures 2a, 2b we plot the coefficients of the various initial conditions A_i^0 , ($i = t, b, \tau, \lambda, \kappa$) at the GUT scale which enter the running A_λ at the electroweak scale ($t = 66$) as a function of Y_λ^0 . Since Y_λ^0 is varied in a large range of values and the illustrations are given for one very small and one very large value of Y_κ^0 , Fig. 2a, 2b cover all four regimes. Furthermore

⁵As far as the numerics are concerned, finite Y_i^0 means $Y_i^0 \sim 10^{-2}$ and infinite Y_i^0 means $Y_i^0 \sim 10^{-1}$.

the initial conditions for $Y_{i=t,b,\tau}^0$ are fixed to the (large) common value 0.1. Thus when Y_λ^0 runs from 0 to 5 one observes the transition from regime 1 to regime 3 in Fig. 2a and from regime 2 to regime 4 in Fig. 2b. [We should keep in mind that the very large values taken by Y_λ^0 or Y_κ^0 are only for the sake of numerical comparison in the deep IRQFP regimes at the electroweak scale. At much higher scales they eventually lead to perturbatively non reliable results.] The trend of the plots is in perfect agreement with what is anticipated from Eq.(D.8). In Fig. 2a one retrieves the coefficients of regime 1 in the region of small Y_λ^0 . For large Y_λ^0 all the coefficients vanish asymptotically as expected in regime 3. [The asterisks (*) at the right of the graph represent the values of the coefficients for $Y_\lambda^0 = 10$.] In Fig 2b where Y_κ^0 is taken very large ($= 20$) we observe the evolution as a function of Y_λ^0 from the regime 2, to the regime 4. The weaker sensitivity to Y_λ^0 (as compared to Fig. 2a) corroborates here the fact that the two regimes 2, 4 lead to identical coefficients in this case, see Eq.(D.8).

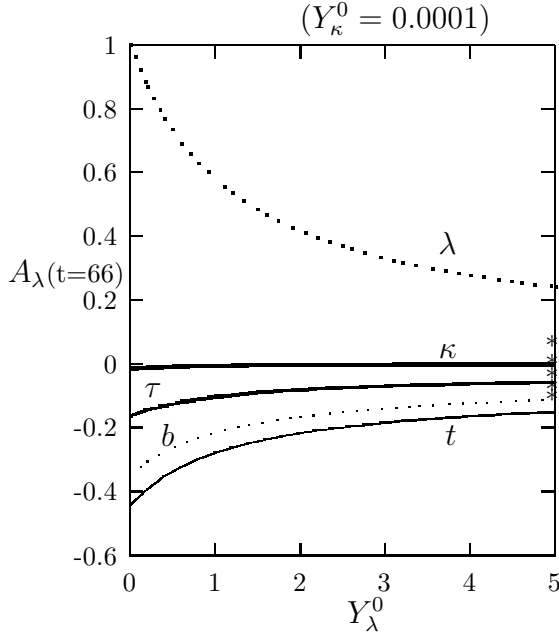


Fig. 2.a

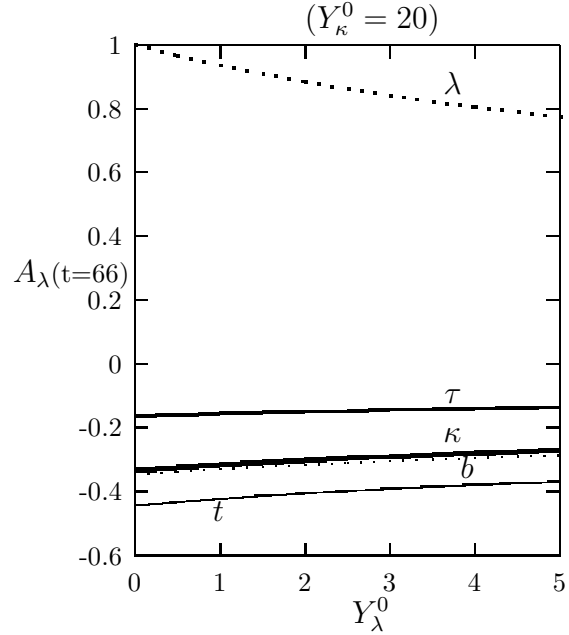


Fig. 2.b

We have made the same analysis for the soft terms Σ_i and m_i and obtained numerical results in total agreement with our analytical expression (Appendix D). We just present here in Fig. 3a, 3b, respectively the dependence of $(m_Q)^2$ and $(m_1)^2$ on the initial GUT scale values of all soft squared masses (not including the trace contribution of Eq.(3.30), for a small value of Y_κ^0 (0.001), and for Y_λ^0 running from 0 to 5. One sees the transition from regime 1 to regime 3 with increasing Y_λ^0 . The dependence on the soft mass $(m_S^0)^2$ vanishes towards regime 1.

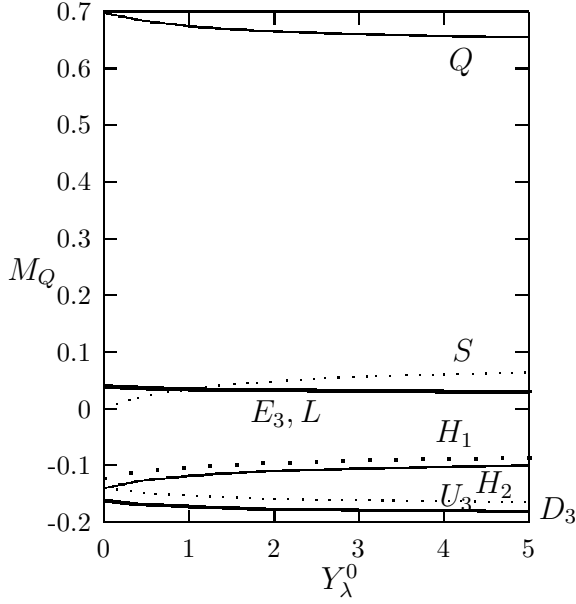


Fig. 3.a

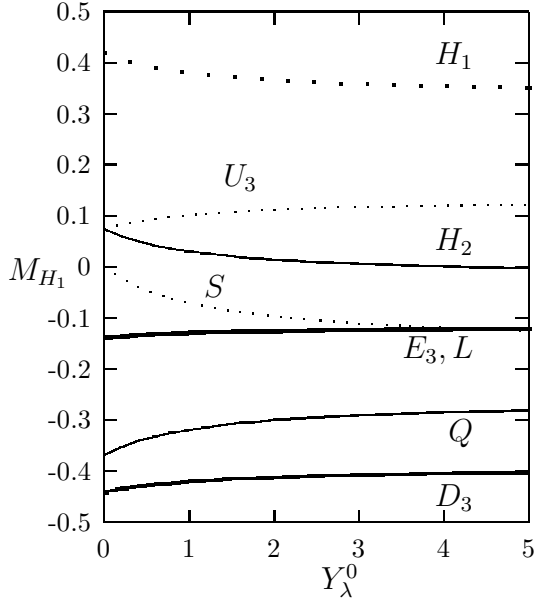


Fig. 3.b

Finally we illustrate in Fig.4 the behaviour of Y_λ, Y_κ in all four regimes, to stress the fact that these regimes do set in effectively, well before that the initial conditions Y_λ^0, Y_κ^0 become infinitely large.

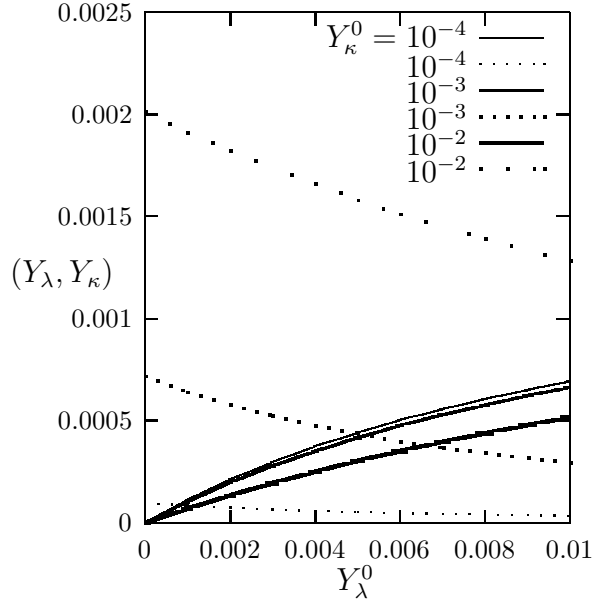


Fig. 4: Y_λ (full lines), Y_κ (dotted lines)

5 Electroweak symmetry breaking

In this section we consider the impact of the four IRQFP regimes on more phenomenological issues. It turns out, as we will show hereafter, that regimes 2 and 4 are consistent

with the requirement of EWSB only for very small $|\hat{\mu}|$. These regimes are thus disfavored or excluded from present experimental exclusion lower bounds on the lightest chargino mass [22, 23]. Regimes 1 and 3 do not suffer from such features and correspond to viable configurations of least sensitivity to initial conditions.

The study is carried out at the tree-level. [Some of the conclusions will remain true if one-loop corrections are included, but we will not discuss the issue further in the present paper.] The EWSB conditions involving the three Higgs v.e.v.'s are obtained from Eq.(2.3) in the form:

$$h_1 [m_1^2 + \lambda^2(h_2^2 + s^2) + \frac{1}{2}\bar{g}^2(h_1^2 - h_2^2) + (A_\lambda\lambda s + \kappa\lambda s^2)\frac{h_2}{h_1}] = 0 \quad (5.1)$$

$$h_2 [m_2^2 + \lambda^2(h_1^2 + s^2) + \frac{1}{2}\bar{g}^2(h_2^2 - h_1^2) + (A_\lambda\lambda s + \kappa\lambda s^2)\frac{h_1}{h_2}] = 0 \quad (5.2)$$

$$s [m_s^2 + \lambda^2(h_1^2 + h_2^2) + 2\kappa^2 s^2 + A_\kappa\kappa s + (\frac{A_\lambda}{s} + 2\kappa)\lambda h_1 h_2] = 0 \quad (5.3)$$

where

$$< H_1 > \equiv \begin{pmatrix} h_1 \\ 0 \end{pmatrix} \quad < H_2 > \equiv \begin{pmatrix} 0 \\ h_2 \end{pmatrix} \quad < S > \equiv s \quad (5.4)$$

Here h_1, h_2 and s are chosen to be real-valued and positive. (see for instance [4] for a discussion of the freedom in the choice of the various parameters).

In addition to Eq.(5.1 - 5.3) the requirement of correct electroweak scale reads

$$h_1^2 + h_2^2 = \frac{M_Z^2}{\bar{g}^2} \quad (5.5)$$

From equations (5.1), (5.2) and (5.5), and provided that non of the v.e.v.'s is vanishing, one easily gets

$$h_1^2 = \frac{M_Z^2}{\bar{g}^2} \frac{\frac{1}{2}M_Z^2 + m_2^2 + \lambda^2 s^2}{M_Z^2 + m_1^2 + m_2^2 + 2\lambda^2 s^2} \quad (5.6)$$

$$h_2^2 = \frac{M_Z^2}{\bar{g}^2} \frac{\frac{1}{2}M_Z^2 + m_1^2 + \lambda^2 s^2}{M_Z^2 + m_1^2 + m_2^2 + 2\lambda^2 s^2} \quad (5.7)$$

$$h_1 h_2 = -\frac{M_Z^2}{\bar{g}^2} \frac{(A_\lambda\lambda s + \kappa\lambda s^2)}{m_1^2 + m_2^2 + \lambda^2(\frac{M_Z^2}{\bar{g}^2} + 2s^2)} \quad (5.8)$$

Note that the above equations yield the familiar EWSB conditions of the MSSM when one goes to the limit $\lambda, \kappa \rightarrow 0, s \rightarrow \infty$ with $\lambda s, \kappa s$ finite, with the identifications $\hat{\mu} \equiv \lambda s, B \equiv A_\lambda + \kappa s, \tan \beta \equiv h_2/h_1, h_1 h_2 \equiv M_Z^2 \sin 2\beta / \bar{g}^2$ (of course with the proviso of Eq.(5.3) which correlates dynamically s , thus $\hat{\mu}$, to the other parameters).

From Eq.(5.8), one obtains⁶

$$\frac{\kappa}{\lambda} = a + b\lambda^2 \quad (5.9)$$

with

$$a = -\frac{A_\lambda}{\hat{\mu}} + \frac{1}{2} \frac{M_Z^2}{\hat{\mu}^2} s_{2\beta} \left(1 - \frac{1/2 M_Z^2 + m_1^2 + \hat{\mu}^2 y_t^2}{m_t^2} \frac{1}{\bar{g}^2}\right) \quad (5.10)$$

$$= -\frac{A_\lambda}{\hat{\mu}} - \frac{1}{t_\beta} - \frac{1}{2t_\beta \hat{\mu}^2} (c_{2\beta} M_Z^2 + 2m_1^2) \quad (5.11)$$

$$b = -\frac{1}{2} \frac{M_Z^2}{\hat{\mu}^2} \frac{s_{2\beta}}{\bar{g}^2} \quad (5.12)$$

Equation (5.9) shows a linear correlation between κ and λ (in the regime of small $\lambda (\ll \bar{g})$) even at low energy, the coefficient a depending on y_t . This was noticed numerically in [20] for λ^0 and κ^0 in constrained cases (universality, $y_t \gg y_b \dots$).

On the other hand, the scalar potential at the electroweak symmetry breaking minimum is obtained from Eqs.(2.3), (5.1-5.3) and reads

$$\begin{aligned} V_{min} = & -(\kappa s + \lambda h_1 h_2)^2 - \lambda^2 s^2 (h_1^2 + h_2^2) \\ & - A_\lambda \lambda s h_1 h_2 - \frac{1}{3} A_\kappa \kappa s^3 - \frac{1}{4} M_Z^2 (h_1^2 + h_2^2) c_{2\beta} \end{aligned} \quad (5.13)$$

Eqs.(5.6 – 5.13) will be very useful when discussing the implication of IRQFP regimes.

As previously stressed, although the IRQFP regimes correspond, strictly speaking, to some or all of the Yukawa couplings taking infinite values at some high energy scale, in practice we stay away from these unphysical configurations. Still, the typical IRQFP is essentially preserved as can be seen from Fig.4, and gives a very good idea of the sensitivity to initial conditions.

Bearing this in mind, we thus refer in the following to regime 1 as implying $\kappa, \lambda \ll \bar{g}$, to regimes 2,4 as implying $\lambda \ll \kappa, \bar{g}$ and to regime 3 as $\kappa \ll \lambda, \bar{g}$ at the electroweak scale, rather than the more strict behavior given in Eqs.((3.15), (3.16)). Since the discussion may get a little too involved, especially in regime 3, the reader interested only in the conclusions can go directly to the summaries in 5.2 and 5.4.

5.1 Regimes 1, 2, 4:

Note first that Eqs.(5.6, 5.7) do not have a solution if $\lambda = 0$ with s finite, unless $m_1^2 + m_2^2$ is fine-tuned to zero in which case there is an infinitely degenerate set (a valley) of solutions for h_1, h_2 . This degeneracy is not lifted by one-loop corrections as can be seen for instance from the form of the top/stop contributions [10] and should thus be discarded as non physical. We now take λ non vanishing but small enough to allow a reliable expansion

⁶ we use from now on the shorthand notation, $t_\beta, s_{2\beta}, c_{2\beta}$ for $\tan \beta, \sin 2\beta, \cos 2\beta$.

to first order. Moreover, we consider separately the cases (a) $s \lesssim m, A$ and (b) $s \gg m, A$ where m, A denote generically the magnitudes of the soft masses and couplings. We will also denote by M_{soft} a generic value of the soft masses and couplings, or the smallest of these values.

(a) $s \lesssim m, A$:

In this case $|\lambda|s \ll m, A$ and an expansion in small parameter is reliable. Performing this expansion in Eqs.(5.6 – 5.8) to second order in $\hat{\mu}/M_{soft}$ (and first order in ϵ) and eliminating h_1, h_2 , the equation determining dynamically $\hat{\mu}(\equiv \lambda s)$ takes the simple form:

$$\zeta \hat{\mu}^2 + \frac{A_\lambda}{m_1^2 + m_2^2} (1 - \epsilon) \hat{\mu} + \frac{\sqrt{(\frac{1}{2}M_Z^2 + m_1^2)(\frac{1}{2}M_Z^2 + m_2^2)}}{|M_Z^2 + m_1^2 + m_2^2|} + O((\frac{\hat{\mu}}{M_{soft}})^3) = 0 \quad (5.14)$$

where

$$\epsilon = \frac{\lambda^2}{\bar{g}^2} \frac{M_Z^2}{m_1^2 + m_2^2} \quad (5.15)$$

$$\begin{aligned} \zeta = & \frac{(m_1^2 - m_2^2)^2}{\sqrt{(M_Z^2 + 2m_1^2)(M_Z^2 + 2m_2^2)} |M_Z^2 + m_1^2 + m_2^2| (M_Z^2 + m_1^2 + m_2^2)} \\ & + \frac{1}{(m_1^2 + m_2^2)} \frac{\kappa}{\lambda} (1 - \epsilon) \end{aligned} \quad (5.16)$$

regimes 2, 4:

Here $|\lambda| \ll |\kappa|, \bar{g}$. Solving Eq.(5.14) for $\hat{\mu}$ leads in this case to

$$\hat{\mu} = \pm \sqrt{-\frac{\lambda}{\kappa} (m_1^2 + m_2^2)} \sqrt{\frac{\sqrt{(\frac{1}{2}M_Z^2 + m_1^2)(\frac{1}{2}M_Z^2 + m_2^2)}}{|M_Z^2 + m_1^2 + m_2^2|}} + O(\frac{\lambda}{\kappa}) \quad (5.17)$$

which, apart from the consistency conditions $(\frac{1}{2}M_Z^2 + m_1^2)(\frac{1}{2}M_Z^2 + m_2^2) \geq 0$ and $\lambda\kappa(m_1^2 + m_2^2) \leq 0$, shows that $|\hat{\mu}|$ tends to be generically very small being suppressed by the size of $\sqrt{|\lambda/\kappa|}$. Thus, even if one chooses the soft parameters sufficiently larger than the electroweak scale, so that the condition $|\lambda|s \ll m, A$ dictated by regimes 1, 2, 4 does not imply *a priori* small $|\hat{\mu}|$ in comparison to the electroweak scale, then the dynamics of regimes 2, 4 will still lead to a vanishing $\hat{\mu}$. A vanishing $\hat{\mu}$ implies a lightest chargino to be lighter than M_W and drops even much lower, for $\tan \beta > 1$, see Eq.(5.38). Such a configuration would be excluded, or at most marginally acceptable, by the LEP II lower bounds on the lightest chargino [22, 23], were it not for the fact that, since λ is small in the regimes under consideration, unconventional

signatures due to displaced vertices can emerge in the case of the (M+1)SSM thus requiring a reanalysis of the experimental data [9]. Even so, the theoretical upper bound (5.38) with vanishing $\hat{\mu}$ will start conflicting with the conservative LEPI kinematical limit of $M_Z/2$ as soon as $\tan\beta > 2.27$ or so.

regime 1:

This regime has a crucial difference with 2 and 4, namely that κ and λ can be generally of the same order. In this case the behavior given in Eq.(5.17) is no more valid. One can of course still solve readily Eq.(5.14) keeping in mind that $|\hat{\mu}| \ll m, A$ in the regime under consideration. Since the $\hat{\mu}$ independent term in Eq.(5.14) is generically of order 1/2 it is straightforward to see, taking into account Eqs.(5.15, 5.16) that a consistent $\hat{\mu}$ requires that A_λ be very large compared to $\sqrt{m_1^2 + m_2^2}$, at the electroweak scale. This suggests that, generically (i.e discarding fine-tuned cancellations in $m_1^2 + m_2^2$), regime 1 disfavors solutions with $s \lesssim m, A$ if the relevant soft parameters are of the same order of magnitude at some initial scale.

(b) $s \gg m, A$:

In this case $|\lambda|s \gtrsim m, A$.

regimes 2,4:

Eq.(5.8) takes the form

$$h_1 h_2 = -\frac{\kappa}{\lambda} \hat{\mu}^2 \frac{M_Z^2}{\bar{g}^2} \frac{1}{(m_1^2 + m_2^2 + 2\hat{\mu}^2)^2} + O(\lambda^0) \quad (5.18)$$

that is, $h_1 h_2$ can be made arbitrarily large in the deep 2,4 regions ($\frac{\lambda}{\kappa} \rightarrow 0$). This is however in contradiction with the behavior dictated by Eqs.(5.6, 5.7) in the same regions and leads to an inconsistency, even if $M_Z^2 + m_1^2 + m_2^2 + 2\hat{\mu}^2$ is artificially fine-tuned to $O(\lambda)$.

regime 1:

Here no simple expressions can be derived and one would have to solve the full-fledged higher order polynomial equation for $\hat{\mu}$ combined from Eqs.(5.6– 5.8).

5.2 Summary for regimes 1, 2, 4

We have shown that the IRQFP regimes 2,4 can be consistent with EWSB only when s is of the order of the soft masses, in which case $|\hat{\mu}|$ becomes too small to be consistent with present limits on chargino masses (or at best marginally consistent if $\tan\beta \lesssim 2.3$ when only conservative LEPI limits are considered). On the other hand we found that for dynamical reasons, and independently of any phenomenological considerations, regime 1 can be consistent only for large s ($\gg m, A$). This last point is one ingredient for the explanation of the numerically established large values of singlet Higgs v.e.v., [10]. We

will come back later to these features. Let us also note that the above conclusions were drawn without taking into account Eq.(5.3). This equation can be viewed in this context as nearly correlating the two extra soft parameters m_s, A_κ and enters the game as a further constraint translated to initial conditions at some high energy scale.

5.3 Regime 3:

Here $|\kappa| \ll |\lambda|, \bar{g}$ and we consider as before two regimes for s . However the discussion will be much more involved. We give hereafter the main steps.

(a) $s \lesssim m, A$:

Since $|\kappa|s \ll m, A$ in this case, it is reliable to expand in the small parameter $\kappa s/M_{soft}$. Adding up Eq.(5.1) divided by h_1 to Eq.(5.2) divided by h_2 and using Eqs.(5.3, 5.5) to eliminate the combinations $h_1 h_2$ and $h_1^2 + h_2^2$ on finds

$$2\hat{\mu}^2 - x\hat{\mu} - \eta + O((\frac{\kappa s}{M_{soft}})^2) = 0 \quad (5.19)$$

with

$$\begin{aligned} x &= \frac{\lambda\kappa}{\bar{g}^2} \frac{M_Z^2 A_\lambda^2}{m_s^2 + \frac{\lambda^2}{\bar{g}^2} M_Z^2} \left(\frac{3}{A_\lambda} - \frac{A_\kappa}{m_s^2 + \frac{\lambda^2}{\bar{g}^2} M_Z^2} \right) \\ \eta &= \frac{\lambda^2}{\bar{g}^2} \left(\frac{A_\lambda^2}{m_s^2 + \frac{\lambda^2}{\bar{g}^2} M_Z^2} - 1 \right) M_Z^2 - m_1^2 - m_2^2 \end{aligned} \quad (5.20)$$

Thus

$$\hat{\mu} = \pm \sqrt{\frac{\eta}{2}} + O(x) \quad (5.21)$$

provided $\eta \gtrsim 0$.

On the other hand, Eq.(5.9) yields

$$\left(\frac{1}{t_\beta} + \frac{\kappa}{\lambda} \right) \hat{\mu}^2 + A_\lambda \hat{\mu} + \frac{1}{2} \left(\frac{c_{2\beta} M_Z^2 + 2m_1^2}{t_\beta} + \frac{\lambda^2}{\bar{g}^2} s_{2\beta} M_Z^2 \right) = 0 \quad (5.22)$$

Consistency between equations (5.22) and (5.19) requires

$$\hat{\mu} = -\frac{1}{2t_\beta A_\lambda} (2m_1^2 + \eta + (c_{2\beta} + \frac{\lambda^2}{\bar{g}^2} (1 - c_{2\beta})) M_Z^2) + O(x, \frac{\kappa}{\lambda}) \quad (5.23)$$

on top of Eq.(5.21). Furthermore, combining Eqs.(5.6, 5.7) to retrieve the familiar EWSB relation

$$\frac{M_Z^2}{2} = \frac{(m_1^2 + \hat{\mu}^2) - (m_2 + \hat{\mu}^2)t_\beta^2}{t_\beta^2 - 1} \quad (5.24)$$

and using Eq.(5.21) one gets a further correlation between m_1, m_2, A_λ and m_s in the form

$$m_1^2 - m_2^2 = ((\frac{\lambda^2}{\bar{g}^2} - 1)M_Z^2 - X)c_{2\beta} \quad (5.25)$$

where

$$X \equiv \frac{\frac{\lambda^2}{\bar{g}^2}M_Z^2}{m_s^2 + \frac{\lambda^2}{\bar{g}^2}M_Z^2}A_\lambda^2 \quad (5.26)$$

Finally, equating the values $\hat{\mu}^2$ from Eqs.(5.21, 5.23) and using Eq.(5.25) to eliminate m_2 one finds the necessary correlation among the mass parameters m_1, m_s, A_λ and M_Z , for given $t_\beta (\equiv \frac{h_2}{h_1})$,

$$\frac{X}{A_\lambda^2} = \frac{t_\beta^2}{1 - c_{2\beta}}(1 \pm \sqrt{1 - \frac{x_1}{A_\lambda^2}}) \equiv \mathcal{R}^\pm \quad (5.27)$$

$$A_\lambda^2 \geq x_1 \quad (5.28)$$

where

$$x_1 \equiv \frac{2}{t_\beta^2}(2m_1^2 + (\frac{\lambda^2}{\bar{g}^2}(1 - c_{2\beta}) + c_{2\beta})M_Z^2) \quad (5.29)$$

Equation (5.27) is easily obtained by eliminating $\hat{\mu}$ between Eqs.(5.21) and (5.23) and solving for the resulting quadratic equation in X . (For simplicity, we skip from now on the explicit indication that all the relations are valid only to zeroth order in small parameters like $\kappa/\lambda, \kappa/\bar{g}, \kappa s/M_{soft}$.)

Incidentally, we note here that $\eta \gtrsim 0$ as required by Eq.(5.21), is automatically implied by the correlations Eqs.(5.25, 5.27). Furthermore, $m_s^2 > 0$ implies immediately

$$0 \leq \mathcal{R}^\pm \leq 1 \quad (5.30)$$

from (5.26) and (5.27). For definiteness we stick hereafter to the phenomenologically likely case $t_\beta > 1$. Then, it is easy to see from Eq.(5.27) that $\mathcal{R}^+ < 1$ is excluded by $t_\beta > 1$, while \mathcal{R}^- is acceptable and leads through Eq.(5.30) to the constraint

$$A_\lambda^2 \geq \frac{x_1}{s_{2\beta}^2} \quad (5.31)$$

which overpowers Eq.(5.28).

One can now determine uniquely $\hat{\mu}$ from Eqs.(5.23, 5.20, 5.25, 5.27):

$$\hat{\mu} = -\frac{1 - c_{2\beta}}{2t_\beta} \frac{X}{A_\lambda} = -\frac{1}{2} t_\beta A_\lambda \left[1 - \sqrt{1 - \frac{x_1}{A_\lambda^2}} \right] \quad (5.32)$$

Now as far as x_1 (Eq.(5.29)) remains positive at the electroweak scale (which is the case if $\tan \beta$ does not become extremely large leading to substantially negative m_1^2 at this same scale), equation (5.32) shows that $|\hat{\mu}|$ is a decreasing function of $|A_\lambda|$. Then using Eq.(5.31) one gets the following upper bound on $\hat{\mu}^2$

$$\hat{\mu}^2 \leq \frac{x_1}{4} \quad (5.33)$$

This constraint leads to an upper bound on the physical lightest chargino mass, which may or may not be consistent with the experimental lower bounds. To go further in this issue, let us include first the requirement that the electroweak symmetry breaking extremum of the scalar potential should indeed be a minimum lower than the $SU(2)_L \times U(1)_Y$ symmetric one at vanishing scalar fields, i.e. that

$$V_{min} < 0 \quad (5.34)$$

Evaluating Eq.(5.13) at $\kappa \simeq 0$ and $\hat{\mu}$ as given by Eq.(5.32), the above condition leads to

$$4t_\beta^2(1 + t_\beta^2)(x_2 - (1 + t_\beta^2)x_1)A_\lambda^2 + (x_2 - (1 + t_\beta^2)x_1)^2 > 0 \quad (5.35)$$

where x_1 is as given before, (Eq.(5.29),

$$x_2 = 4\left(\frac{\lambda^2}{\bar{g}^2} + \frac{c_{2\beta}^2}{s_{2\beta}^2}\right)M_Z^2 \quad (5.36)$$

and Eq.(5.31) has been implicitly used. As can be seen from Eq.(5.35), this equation leads to a constraint only if $x_2 - (1 + t_\beta^2)x_1 < 0$. We need to consider two cases:

i) $x_2 - (1 + t_\beta^2)x_1 > 0$: in this case Eq.(5.34) is trivially verified, but Eq.(5.33) together with $x_2 - (1 + t_\beta^2)x_1 > 0$ and (5.36) lead to

$$\hat{\mu}^2 \leq \frac{1 + t_\beta^2}{4t_\beta^2} \left(\frac{\lambda^2}{\bar{g}^2} s_{2\beta}^2 + c_{2\beta}^2 \right) M_Z^2 \quad (5.37)$$

Since the chargino sector is identical to that of the MSSM one can study directly the effect of the above bound on the mass of the lightest chargino denoted hereafter by $M_{\chi_1^\pm}$. Using a rigorous upper bound on $M_{\chi_1^\pm}$

$$M_{\chi_1^+} \leq \sqrt{\hat{\mu}^2 + \frac{2M_W^2}{1+t_\beta^2}} \quad (5.38)$$

in conjunction with Eq.(5.37) one gets immediately a $\tan \beta$ dependent (and $\hat{\mu}$ independent) upper bound on $M_{\chi_1^+}$. Confronting this upper bound with the present experimental lower bounds from the LEP II exclusion analyses [22, 23] one finds typically that for $\tan \beta > 1$ only a small range of $\tan \beta$ values close to 1 are possible. For example, taking $\lambda = \bar{g}$ one has

$$M_{\chi_1^+} \leq \sqrt{\frac{2}{1+t_\beta^2} M_W^2 + \frac{1+t_\beta^2}{4t_\beta^2} M_Z^2} \quad (5.39)$$

Comparing for instance with the experimental analysis of [23] (Fig 5 therein) one finds that only a very small range, $1 < \tan \beta < 1.3 - 1.6$, is consistent with our regime in this case. Of course one should keep in mind the model dependence of the experimental analyses. However we should stress that since in the regime under consideration λ is relatively large ($\sim \bar{g}$), we are in a configuration which is safe from significant unconventional signals due to displaced vertices that can occur in the (M+1)SSM [9]. The comparison with MSSM-based experimental analyses is thus fully consistent here.

ii) $x_2 - (1+t_\beta^2)x_1 < 0$: In this case the stability of the electroweak symmetry breaking minimum requires that A_λ^2 be bounded from above at the electroweak scale. This upper bound \bar{A}_λ^2 can be easily read from Eq.(5.35)⁷. However, $A_\lambda^2 < \bar{A}_\lambda^2$ will lead, through Eqs.(5.27, 5.26, 5.29, 5.36) to an upper bound on m_s^2 (remember that Eq.(5.27) is valid for \mathcal{R}^- , see the discussion following Eq.(5.30)). Working out the algebra one finds

$$m_s^2 < \frac{\lambda^2}{\bar{g}^2} \frac{M_Z^4}{4m_1^2 - (t_\beta^2 - 1)M_Z^2} \frac{t_\beta^2}{(1+t_\beta^2)^2} ((t_\beta^2 - 1)^2 + 4t_\beta^2 \frac{\lambda^2}{\bar{g}^2}) \quad (5.40)$$

This inequality shows an anti-correlation between m_s^2 and m_1^2 which suggests that universality assumptions between the singlet and doublet soft scalar masses may be disfavoured. To see this for any value of t_β one should plug the running expressions for m_1^2, m_s^2 given in Eqs.((B.18), (B.20)) and evaluate the auxiliary functions given in Eqs.((3.5) – (3.9)) and in Appendix B (approximating them for instance to their first order iteration). Instead, let us give here a simpler and more qualitative argument. It is clear from Eqs.(5.33, 5.37) and from the dependence of x_1 on t_β (Eq.(5.29)) that the higher the experimental exclusion bound on $M_{\chi_1^+}$ the less favoured are the

⁷ which in turn leads to a phenomenologically harmless lower bound on $|\hat{\mu}|$, $|\hat{\mu}| > |\hat{\mu}(\bar{A}_\lambda)|$, rather than an improved upper bound like in i).

large t_β configurations. This conclusion is reinforced by the fact that for a given initial condition m_0^2 , the running m_1^2 decreases faster for larger t_β , thus driving x_1 to small values. So let us concentrate only on the small t_β region. In this case the running m_s^2 and m_1^2 are well approximated by the simple analytical solutions given in [11], namely $m_1^2 \simeq m_0^2 + M_0^2/2$, $m_s^2 \simeq m_0^2$, in the vicinity of the infrared effective fixed point for small $\tan\beta$ (~ 1) and assuming universal initial conditions for the scalar and fermion soft masses (denoted respectively by m_0 and M_0). Using these relations in Eq.(5.40) one finds

$$m_0^2 < \frac{1}{2}\left(-\frac{M_0^2}{2} + \sqrt{\frac{M_0^4}{4} + M_Z^4}\right) + O(t_\beta - 1) < \frac{M_Z^2}{2} + O(t_\beta - 1) \quad (5.41)$$

that is m_0^2 is forced to be rather small (eventually vanishing) for large M_0^2 , the latter being required by experimental lower bounds on $M_{\chi_1^+}$. A way to avoid such an anti-correlation is to relax the universality between the singlet and doublet Higgs soft masses.

(b) $s \gg m, A$:

In this case, no generic statement can be made about the size of κs . If the magnitude of λ is such that $M_{soft}/(\lambda s)$ is small then Eqs.(5.6, 5.7) can be cast in the form

$$h_1^2 = \frac{M_Z^2}{2\bar{g}^2}\left[1 + \frac{m_2^2 - m_1^2}{2\hat{\mu}^2} + O\left(\left(\frac{M_{soft}}{\hat{\mu}}\right)^4\right)\right] \quad (5.42)$$

$$h_2^2 = \frac{M_Z^2}{2\bar{g}^2}\left[1 + \frac{m_1^2 - m_2^2}{2\hat{\mu}^2} + O\left(\left(\frac{M_{soft}}{\hat{\mu}}\right)^4\right)\right] \quad (5.43)$$

Feeding the above equations back in Eq.(5.1) one gets,

$$\hat{\mu}^2 + \frac{\gamma}{\hat{\mu}} + \delta + O\left(\frac{\kappa}{\lambda}\left(\frac{M_{soft}}{\hat{\mu}}\right)^2, \left(\frac{M_{soft}}{\hat{\mu}}\right)^4\right) \times \frac{M_Z^2}{2\bar{g}^2} = 0 \quad (5.44)$$

where

$$\gamma = \frac{1}{2}A_\lambda(m_1^2 - m_2^2) \quad (5.45)$$

$$\delta = \frac{\lambda^2}{\bar{g}^2}M_Z^2 + m_1^2 + \frac{\kappa}{2\lambda}(m_1^2 - m_2^2) \quad (5.46)$$

In the deep regime 3, $|\kappa/\lambda| \ll 1$, thus δ is positive (except for very large t_β where m_1^2 can become negative at the electroweak scale). It then follows from Eq.(5.44) that this regime can not be dynamically consistent in the case at hand, *i.e.* as far as λ is not too small in this regime so that $s \gg m, A$ implies very large $|\lambda|s(= \hat{\mu})$.

Finally let us stress that Eq.(5.44) is more general than in the context of regime 3 (the condition $\kappa \ll \bar{g}$ was not used in establishing this equation). Furthermore, $|\delta|$ is by definition of order M_{soft}^2 whatever its sign. Thus if s happens to be extremely large compared to the soft masses, then Eq.(5.44) forbids $\hat{\mu}$ to be very large too, i.e. λ cannot be large ($\sim \bar{g}$ say). We thus have a further ingredient in understanding purely numerical studies were indeed very large s required very small λ (see for instance table 1 of reference [11]).

5.4 Summary for regime 3

Regime 3 is the trickiest. We have shown that an s much larger than the soft parameters is forbidden unless the hierarchy $|\kappa| \ll |\lambda| \lesssim M_{soft}/s$ is realized. On the other hand, in the case where s is of the order of the soft parameters the stability of the EWSB vacuum has to be invoked. In configurations where this stability is automatic, $\hat{\mu}$ leads to light charginos inconsistent with the present experimental limits (except for a small window $1 < \tan \beta < 1.3 - 1.6$ which will also be closed by a few GeV improvement of these limits). When the stability of the EWSB vacuum is not automatic, then the generic price to pay is small $\tan \beta$ values and either a relaxation of universality between the singlet and doublet Higgs soft scalar masses, or a small universal soft scalar mass m_0 anti-correlated with a large universal soft gaugino mass M_0 to ensure consistency with experimental limits on the lightest chargino. Clearly future improvement of these mass limits will reinforce the above conclusions.

6 Conclusion

We have made an extended analytical study of the scale evolution of the various basic parameters, as well as of the spontaneous electroweak symmetry breaking and the dynamical determination of the $\hat{\mu}$ parameter in the (M+1)SSM. In particular, we identified four regimes of effective infrared fixed points behaviour corresponding to various relative magnitudes of the two couplings λ and κ that enter the gauge singlet scalar sector. These regimes correspond to the configurations of least sensitivity to the initial (GUT-scale) conditions of most of the parameters. We have determined analytically this sensitivity and shown how it generalizes the MSSM case. Furthermore, the analysis of electroweak symmetry breaking (which did not require any numerical scan over the parameter space) showed that some of these regimes, taken in a wider sense, are generically excluded by negative experimental searches for charginos or by purely dynamical considerations, and that the others lead to very large singlet vacuum expectation values which can be reduced, however, if some amount of non-universality of the soft parameters is allowed.

The general analytical solutions given in this paper do not rely on any model-dependent GUT-scale assumptions. In practice they lead to approximate solutions to the RGE's (with controllable convergence), in analytically closed forms and for any value of the $\tan \beta$

parameter. They thus allow a precise study of the dynamical properties of the (M+1)SSM. Besides, they are readily generalizable to further extensions of the Higgs sector. This provides a basis to study the genericity of these properties beyond the (M+1)SSM and to gain a more thorough understanding of the sensitivity of the Higgs sector phenomenology to specific underlying supersymmetric models.

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Appendix A: The coefficients a_{ki} , b_i and c_{ki}

In this appendix we define the coefficients a_{ki} and c_{ki} introduced in the RGE (3.1)

$a_{tt} = 6$	$a_{bt} = 1$	$a_{\tau t} = 0$	$a_{\lambda t} = 3$	$a_{\kappa t} = 0$
$a_{tb} = 1$	$a_{bb} = 6$	$a_{\tau b} = 3$	$a_{\lambda b} = 3$	$a_{\kappa b} = 0$
$a_{t\tau} = 0$	$a_{b\tau} = 1$	$a_{\tau\tau} = 4$	$a_{\lambda\tau} = 1$	$a_{\kappa\tau} = 0$
$a_{t\lambda} = 1$	$a_{b\lambda} = 1$	$a_{\tau\lambda} = 1$	$a_{\lambda\lambda} = 4$	$a_{\kappa\lambda} = 6$
$a_{t\kappa} = 0$	$a_{b\kappa} = 0$	$a_{\tau\kappa} = 0$	$a_{\lambda\kappa} = 2$	$a_{\kappa\kappa} = 6$

$b_1 = 11$	$b_2 = 1$	$b_3 = -3$
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$c_{t1} = \frac{13}{9}$	$c_{b1} = \frac{7}{9}$	$c_{\tau 1} = 3$	$c_{\lambda 1} = 1$	$c_{\kappa 1} = 0$
$c_{t2} = 3$	$c_{b2} = 3$	$c_{\tau 2} = 3$	$c_{\lambda 2} = 3$	$c_{\kappa 2} = 0$
$c_{t3} = \frac{16}{3}$	$c_{b3} = \frac{16}{3}$	$c_{\tau 3} = 0$	$c_{\lambda 3} = 0$	$c_{\kappa 3} = 0$

Appendix B: Exact solutions for the A 's and the Σ 's

Hereafter we give the (recursive)-equations defining the auxiliary functions which enter the A 's and Σ 's.

The A 's

$$e_t = \frac{1}{E_t} \frac{d\tilde{E}_t}{d\eta} + Y_b^0 \frac{A_b^0 \int u_b - \int u_b e_b}{1 + 6Y_b^0 \int u_b} + Y_\lambda^0 \frac{A_\lambda^0 \int u_\lambda - \int u_\lambda e_\lambda}{1 + 4Y_\lambda^0 \int u_\lambda} \quad (\text{B.1})$$

$$\begin{aligned}
e_b &= \frac{1}{E_b} \frac{d\tilde{E}_b}{d\eta} + Y_t^0 \frac{A_t^0 \int u_t - \int u_t e_t}{1 + 6Y_t^0 \int u_t} + Y_\tau^0 \frac{A_\tau^0 \int u_\tau - \int u_\tau e_\tau}{1 + 4Y_\tau^0 \int u_\tau} \\
&+ Y_\lambda^0 \frac{A_\lambda^0 \int u_\lambda - \int u_\lambda e_\lambda}{1 + 4Y_\lambda^0 \int u_\lambda}
\end{aligned} \tag{B.2}$$

$$e_\tau = \frac{1}{E_\tau} \frac{d\tilde{E}_\tau}{d\eta} + 3Y_b^0 \frac{A_b^0 \int u_b - \int u_b e_b}{1 + 6Y_b^0 \int u_b} + Y_\lambda^0 \frac{A_\lambda^0 \int u_\lambda - \int u_\lambda e_\lambda}{1 + 4Y_\lambda^0 \int u_\lambda} \tag{B.3}$$

$$\begin{aligned}
e_\lambda &= \frac{1}{E_\lambda} \frac{d\tilde{E}_\lambda}{d\eta} + 3Y_t^0 \frac{A_t^0 \int u_t - \int u_t e_t}{1 + 6Y_t^0 \int u_t} + 3Y_b^0 \frac{A_b^0 \int u_b - \int u_b e_b}{1 + 6Y_b^0 \int u_b} \\
&+ Y_\tau^0 \frac{A_\tau^0 \int u_\tau - \int u_\tau e_\tau}{1 + 4Y_\tau^0 \int u_\tau} + 2Y_\kappa^0 \frac{A_\kappa^0 \int u_\kappa - \int u_\kappa e_\kappa}{1 + 6Y_\kappa^0 \int u_\kappa}
\end{aligned} \tag{B.4}$$

$$e_\kappa = \frac{1}{E_\kappa} \frac{d\tilde{E}_\kappa}{d\eta} + 6Y_\lambda^0 \frac{A_\lambda^0 \int u_\lambda - \int u_\lambda e_\lambda}{1 + 4Y_\lambda^0 \int u_\lambda} \tag{B.5}$$

where the variations of \tilde{E}_k should be taken at $\eta = \bar{\eta} = 0$ and are given by

$$\frac{1}{E_k} \frac{d\tilde{E}_k}{d\eta} = t \sum_{i=1}^3 c_{ki} \alpha_i M_i^0 \tag{B.6}$$

The Σ 's

$$\begin{aligned}
\xi_t &= \frac{1}{E_t} \frac{d^2 \tilde{E}_t}{d\eta d\bar{\eta}} + \frac{2}{E_t} \frac{d\tilde{E}_t}{d\eta} \left[Y_b^0 \frac{A_b^0 \int u_b - \int u_b e_b}{1 + 6Y_b^0 \int u_b} + Y_\lambda^0 \frac{A_\lambda^0 \int u_\lambda - \int u_\lambda e_\lambda}{1 + 4Y_\lambda^0 \int u_\lambda} \right] \\
&- Y_b^0 \frac{(\Sigma_b^0 + (A_b^0)^2) \int u_b - 2A_b^0 \int u_b e_b + \int u_b \xi_b}{1 + 6Y_b^0 \int u_b} \\
&- Y_\lambda^0 \frac{(\Sigma_\lambda^0 + (A_\lambda^0)^2) \int u_\lambda - 2A_\lambda^0 \int u_\lambda e_\lambda + \int u_\lambda \xi_\lambda}{1 + 4Y_\lambda^0 \int u_\lambda} \\
&+ 7(Y_b^0)^2 \left[\frac{A_b^0 \int u_b - \int u_b e_b}{1 + 6Y_b^0 \int u_b} \right]^2 + 5(Y_\lambda^0)^2 \left[\frac{A_\lambda^0 \int u_\lambda - \int u_\lambda e_\lambda}{1 + 4Y_\lambda^0 \int u_\lambda} \right]^2 \\
&+ 2Y_b^0 Y_\lambda^0 \frac{(A_b^0 \int u_b - \int u_b e_b)(A_\lambda^0 \int u_\lambda - \int u_\lambda e_\lambda)}{(1 + 6Y_b^0 \int u_b)(1 + 4Y_\lambda^0 \int u_\lambda)}
\end{aligned} \tag{B.7}$$

$$\xi_b = \frac{1}{E_b} \frac{d^2 \tilde{E}_b}{d\eta d\bar{\eta}} + \frac{2}{E_b} \frac{d\tilde{E}_b}{d\eta} \left[Y_t^0 \frac{A_t^0 \int u_t - \int u_t e_t}{1 + 6Y_t^0 \int u_t} + Y_\tau^0 \frac{A_\tau^0 \int u_\tau - \int u_\tau e_\tau}{1 + 4Y_\tau^0 \int u_\tau} \right]$$

$$\begin{aligned}
& + Y_\lambda^0 \frac{A_\lambda^0 \int u_\lambda - \int u_\lambda e_\lambda}{1 + 4Y_\lambda^0 \int u_\lambda} \Big] \\
& - Y_t^0 \frac{(\Sigma_t^0 + (A_t^0)^2) \int u_t - 2A_t^0 \int u_t e_t + \int u_t \xi_t}{1 + 6Y_t^0 \int u_t} \\
& - Y_\tau^0 \frac{(\Sigma_\tau^0 + (A_\tau^0)^2) \int u_\tau - 2A_\tau^0 \int u_\tau e_\tau + \int u_\tau \xi_\tau}{1 + 4Y_\tau^0 \int u_\tau} \\
& - Y_\lambda^0 \frac{(\Sigma_\lambda^0 + (A_\lambda^0)^2) \int u_\lambda - 2A_\lambda^0 \int u_\lambda e_\lambda + \int u_\lambda \xi_\lambda}{1 + 4Y_\lambda^0 \int u_\lambda} \\
& + 7(Y_t^0)^2 \left[\frac{A_t^0 \int u_t - \int u_t e_t}{1 + 6Y_t^0 \int u_t} \right]^2 \\
& + 5(Y_\tau^0)^2 \left[\frac{A_\tau^0 \int u_\tau - \int u_\tau e_\tau}{1 + 4Y_\tau^0 \int u_\tau} \right]^2 + 5(Y_\lambda^0)^2 \left[\frac{A_\lambda^0 \int u_\lambda - \int u_\lambda e_\lambda}{1 + 4Y_\lambda^0 \int u_\lambda} \right]^2 \\
& + 2Y_t^0 Y_\lambda^0 \frac{(A_t^0 \int u_t - \int u_t e_t)}{(1 + 6Y_t^0 \int u_t)} \frac{(A_\lambda^0 \int u_\lambda - \int u_\lambda e_\lambda)}{(1 + 4Y_\lambda^0 \int u_\lambda)} \\
& + 2Y_t^0 Y_\tau^0 \frac{(A_t^0 \int u_t - \int u_t e_t)}{(1 + 6Y_t^0 \int u_t)} \frac{(A_\tau^0 \int u_\tau - \int u_\tau e_\tau)}{(1 + 4Y_\tau^0 \int u_\tau)} \\
& + 2Y_\tau^0 Y_\lambda^0 \frac{(A_\tau^0 \int u_\tau - \int u_\tau e_\tau)}{(1 + 4Y_\tau^0 \int u_\tau)} \frac{(A_\lambda^0 \int u_\lambda - \int u_\lambda e_\lambda)}{(1 + 4Y_\lambda^0 \int u_\lambda)}
\end{aligned} \tag{B.8}$$

$$\begin{aligned}
\xi_\tau &= \frac{1}{E_\tau} \frac{d^2 \tilde{E}_\tau}{d\eta d\bar{\eta}} + \frac{2}{E_\tau} \frac{d\tilde{E}_\tau}{d\eta} \left[3Y_b^0 \frac{A_b^0 \int u_b - \int u_b e_b}{1 + 6Y_b^0 \int u_b} + Y_\lambda^0 \frac{A_\lambda^0 \int u_\lambda - \int u_\lambda e_\lambda}{1 + 4Y_\lambda^0 \int u_\lambda} \right] \\
& - 3 \frac{(\Sigma_b^0 + (A_b^0)^2) \int u_b - 2A_b^0 \int u_b e_b + \int u_b \xi_b}{1 + 6Y_b^0 \int u_b} \\
& - Y_\lambda^0 \frac{(\Sigma_\lambda^0 + (A_\lambda^0)^2) \int u_\lambda - 2A_\lambda^0 \int u_\lambda e_\lambda + \int u_\lambda \xi_\lambda}{1 + 4Y_\lambda^0 \int u_\lambda} \\
& + 27(Y_b^0)^2 \left[\frac{A_b^0 \int u_b - \int u_b e_b}{1 + 6Y_b^0 \int u_b} \right]^2 + 5(Y_\lambda^0)^2 \left[\frac{A_\lambda^0 \int u_\lambda - \int u_\lambda e_\lambda}{1 + 4Y_\lambda^0 \int u_\lambda} \right]^2 \\
& + 6Y_b^0 Y_\lambda^0 \frac{(A_b^0 \int u_b - \int u_b e_b)}{(1 + 6Y_b^0 \int u_b)} \frac{(A_\lambda^0 \int u_\lambda - \int u_\lambda e_\lambda)}{(1 + 4Y_\lambda^0 \int u_\lambda)},
\end{aligned} \tag{B.9}$$

$$\begin{aligned}
\xi_\lambda &= \frac{1}{E_\lambda} \frac{d^2 \tilde{E}_\lambda}{d\eta d\bar{\eta}} + \frac{2}{E_\lambda} \frac{d\tilde{E}_\lambda}{d\eta} \left[3Y_t^0 \frac{A_t^0 \int u_t - \int u_t e_t}{1 + 6Y_t^0 \int u_t} + 3Y_b^0 \frac{A_b^0 \int u_b - \int u_b e_b}{1 + 6Y_b^0 \int u_b} \right. \\
& + \left. Y_\tau^0 \frac{A_\tau^0 \int u_\tau - \int u_\tau e_\tau}{1 + 4Y_\tau^0 \int u_\tau} + 2Y_\kappa^0 \frac{A_\kappa^0 \int u_\kappa - \int u_\kappa e_\kappa}{1 + 6Y_\kappa^0 \int u_\kappa} \right] \\
& - 3Y_t^0 \frac{(\Sigma_t^0 + (A_t^0)^2) \int u_t - 2A_t^0 \int u_t e_t + \int u_t \xi_t}{1 + 6Y_t^0 \int u_t} \\
& - 3Y_b^0 \frac{(\Sigma_b^0 + (A_b^0)^2) \int u_b - 2A_b^0 \int u_b e_b + \int u_b \xi_b}{1 + 6Y_b^0 \int u_b}
\end{aligned}$$

$$\begin{aligned}
& - Y_\tau^0 \frac{(\Sigma_\tau^0 + (A_\tau^0)^2) \int u_\tau - 2A_\tau^0 \int u_\tau e_\tau + \int u_\tau \xi_\tau}{1 + 4Y_\tau^0 \int u_\tau} \\
& - 2Y_\kappa^0 \frac{(\Sigma_\kappa^0 + (A_\kappa^0)^2) \int u_\kappa - 2A_\kappa^0 \int u_\kappa e_\kappa + \int u_\kappa \xi_\kappa}{1 + 6Y_\kappa^0 \int u_\kappa} \\
& + 27(Y_t^0)^2 \left[\frac{A_t^0 \int u_t - \int u_t e_t}{1 + 6Y_t^0 \int u_t} \right]^2 + 27(Y_b^0)^2 \left[\frac{A_b^0 \int u_b - \int u_b e_b}{1 + 6Y_b^0 \int u_b} \right]^2 \\
& + 5(Y_\tau^0)^2 \left[\frac{A_\tau^0 \int u_\tau - \int u_\tau e_\tau}{1 + 4Y_\tau^0 \int u_\tau} \right]^2 + 16(Y_\kappa^0)^2 \left[\frac{A_\kappa^0 \int u_\kappa - \int u_\kappa e_\kappa}{1 + 6Y_\kappa^0 \int u_\kappa} \right]^2 \\
& + 18Y_t^0 Y_b^0 \frac{(A_t^0 \int u_t - \int u_t e_t)}{(1 + 6Y_t^0 \int u_t)} \frac{(A_b^0 \int u_b - \int u_b e_b)}{(1 + 6Y_b^0 \int u_b)} \\
& + 6Y_t^0 Y_\tau^0 \frac{(A_t^0 \int u_t - \int u_t e_t)}{(1 + 6Y_t^0 \int u_t)} \frac{(A_\tau^0 \int u_\tau - \int u_\tau e_\tau)}{(1 + 4Y_\tau^0 \int u_\tau)} \\
& + 12Y_t^0 Y_\kappa^0 \frac{(A_t^0 \int u_t - \int u_t e_t)}{(1 + 6Y_t^0 \int u_t)} \frac{(A_\kappa^0 \int u_\kappa - \int u_\kappa e_\kappa)}{(1 + 6Y_\kappa^0 \int u_\kappa)} \\
& + 6Y_b^0 Y_\tau^0 \frac{(A_b^0 \int u_b - \int u_b e_b)}{(1 + 6Y_b^0 \int u_b)} \frac{(A_\tau^0 \int u_\tau - \int u_\tau e_\tau)}{(1 + 4Y_\tau^0 \int u_\tau)} \\
& + 12Y_b^0 Y_\kappa^0 \frac{(A_b^0 \int u_b - \int u_b e_b)}{(1 + 6Y_b^0 \int u_b)} \frac{(A_\kappa^0 \int u_\kappa - \int u_\kappa e_\kappa)}{(1 + 6Y_\kappa^0 \int u_\kappa)} \\
& + 4Y_\tau^0 Y_\kappa^0 \frac{(A_\tau^0 \int u_\tau - \int u_\tau e_\tau)}{(1 + 4Y_\tau^0 \int u_\tau)} \frac{(A_\kappa^0 \int u_\kappa - \int u_\kappa e_\kappa)}{(1 + 6Y_\kappa^0 \int u_\kappa)}, \tag{B.10}
\end{aligned}$$

$$\begin{aligned}
\xi_\kappa &= \frac{1}{E_\kappa} \frac{d^2 \tilde{E}_\kappa}{d\eta d\bar{\eta}} + \frac{2}{E_\kappa} \frac{d\tilde{E}_\kappa}{d\eta} \left[6Y_\lambda^0 \frac{A_\lambda^0 \int u_\lambda - \int u_\lambda e_\lambda}{1 + 4Y_\lambda^0 \int u_\lambda} \right] \\
& - 6 \frac{(\Sigma_\lambda^0 + (A_\lambda^0)^2) \int u_\lambda - 2A_\lambda^0 \int u_\lambda e_\lambda + \int u_\lambda \xi_\lambda}{1 + 4Y_\lambda^0 \int u_\lambda} \\
& + 60(Y_\lambda^0)^2 \left[\frac{A_\lambda^0 \int u_\lambda - \int u_\lambda e_\lambda}{1 + 4Y_\lambda^0 \int u_\lambda} \right]^2 \tag{B.11}
\end{aligned}$$

with

$$\frac{1}{E_k} \frac{d^2 \tilde{E}_k}{d\eta d\bar{\eta}} \Big|_{\eta=0, \bar{\eta}=0} = t^2 \left(\sum_{i=1}^3 c_{ki} \alpha_i M_i^0 \right)^2 + 2t \sum_{i=1}^3 c_{ki} \alpha_i (M_i^0)^2 - t^2 \sum_{i=1}^3 c_{ki} \alpha_i^2 (M_i^0)^2 \tag{B.12}$$

The m 's

Due to linear relations [24, 16] which follow from the RG equations (3.1), (3.25) (dropping out momentarily the trace term S in the latter equation), one can express the soft masses for squarks, sleptons and Higgses in terms of the Σ_k in the form

$$m_T^2 = (m_T^0)^2 + \frac{27f_1 - 81f_2}{96} \quad (\text{B.13})$$

$$+ \frac{45(\Sigma_t - \Sigma_t^0) + 3(\Sigma_b - \Sigma_b^0) + 6(\Sigma_\tau - \Sigma_\tau^0) - 27(\Sigma_\lambda - \Sigma_\lambda^0) + 9(\Sigma_\kappa - \Sigma_\kappa^0)}{96}$$

$$m_B^2 = (m_B^0)^2 + \frac{21f_1 - 63f_2}{96} \quad (\text{B.14})$$

$$+ \frac{3(\Sigma_t - \Sigma_t^0) + 45(\Sigma_b - \Sigma_b^0) + 6(\Sigma_\tau - \Sigma_\tau^0) - 21(\Sigma_\lambda - \Sigma_\lambda^0) + 7(\Sigma_\kappa - \Sigma_\kappa^0)}{96}$$

$$m_Q^2 = (m_Q^0)^2 + \frac{-24f_1 + 72f_2}{96} \quad (\text{B.15})$$

$$+ \frac{24(\Sigma_t - \Sigma_t^0) + 24(\Sigma_b - \Sigma_b^0) - 24(\Sigma_\lambda - \Sigma_\lambda^0) + 8(\Sigma_\kappa - \Sigma_\kappa^0)}{96}$$

$$m_L^2 = (m_L^0)^2 + \frac{-33f_1 + 99f_2}{96} \quad (\text{B.16})$$

$$+ \frac{9(\Sigma_t - \Sigma_t^0) - 9(\Sigma_b - \Sigma_b^0) + 30(\Sigma_\tau - \Sigma_\tau^0) - 15(\Sigma_\lambda - \Sigma_\lambda^0) + 5(\Sigma_\kappa - \Sigma_\kappa^0)}{96}$$

$$m_E^2 = (m_E^0)^2 + \frac{30f_1 - 90f_2}{96} \quad (\text{B.17})$$

$$+ \frac{18(\Sigma_t - \Sigma_t^0) - 18(\Sigma_b - \Sigma_b^0) + 60(\Sigma_\tau - \Sigma_\tau^0) - 30(\Sigma_\lambda - \Sigma_\lambda^0) + 10(\Sigma_\kappa - \Sigma_\kappa^0)}{96}$$

$$m_1^2 = (m_1^0)^2 + \frac{3f_1 - 9f_2}{96} \quad (\text{B.18})$$

$$+ \frac{-27(\Sigma_t - \Sigma_t^0) + 27(\Sigma_b - \Sigma_b^0) + 6(\Sigma_\tau - \Sigma_\tau^0) + 45(\Sigma_\lambda - \Sigma_\lambda^0) - 15(\Sigma_\kappa - \Sigma_\kappa^0)}{96}$$

$$m_2^2 = (m_2^0)^2 + \frac{-3f_1 + 9f_2}{96} \quad (\text{B.19})$$

$$+ \frac{27(\Sigma_t - \Sigma_t^0) - 27(\Sigma_b - \Sigma_b^0) - 6(\Sigma_\tau - \Sigma_\tau^0) + 51(\Sigma_\lambda - \Sigma_\lambda^0) - 17(\Sigma_\kappa - \Sigma_\kappa^0)}{96}$$

$$m_s^2 = (m_s^0)^2 + \frac{1}{3}(\Sigma_\kappa - \Sigma_\kappa^0) \quad (\text{B.20})$$

where

$$f_i = \frac{(M_i^0)^2}{b_i} \left(1 - \frac{1}{(1 + b_i \alpha_0 t)^2} \right).$$

To obtain the complete solutions one should add to each of the above equations the corresponding trace term contribution, see Eqs.(3.29), (3.30) and the discussion in section 3.

Appendix C: The Y 's in the IRQFP regimes

We give in this appendix some details about the derivation of Eqs.(3.14) –(3.16). For later use we define

$$r_i = \frac{Y_i^0}{Y_t^0} < \infty \quad (\text{C.1})$$

when it makes sense, namely when Y_t^0 and Y_i^0 go simultaneously to infinity with a fixed ratio r_i .

We will show inductively, in the same spirit as [16], that in such a regime $u_i \rightarrow u_i^\infty$ with

$$u_i^\infty = \frac{u_i^{\text{FP}}}{(Y_t^0)^{p_i}} \quad (\text{C.2})$$

where the p_i 's are fixed numbers which we will explicitly determine, and the u_i^{FP} 's are initial conditions independent and will be defined implicitly through equations (C.6) – (C.10).

To proceed we consider finite order iteration approximations to Eqs.(3.5)– (3.9). To obtain the $(n+1)^{\text{th}}$ order approximation to u_i , denoted $u_i^{(n+1)}$, in terms of the $u_i^{(n)}$, one makes the formal substitutions $u_i \rightarrow u_i^{(n+1)}$ and $u_i \rightarrow u_i^{(n)}$ respectively on the lefthand and righthand sides of Eqs.(3.5)– (3.9). If for any given one of the four IRQFP regimes of section 3.2, the u_i 's have the behaviour of Eq.(C.2) at the n^{th} order, i.e.

$$u_i^{(n)} = \frac{u_i^{\text{FP}(n)}}{(Y_t^0)^{p_i^{(n)}}} \quad (\text{C.3})$$

where the scale dependent functions $u^{\text{FP}(n)}$ are Y_t^0 independent but possibly $r_b, r_\tau, r_\lambda, r_\kappa$ dependent, then the same is true at the $(n+1)^{\text{th}}$ order with the following recursive equation for the p_i 's

$$\begin{pmatrix} p_t^{(n+1)} \\ p_b^{(n+1)} \\ p_\tau^{(n+1)} \\ p_\lambda^{(n+1)} \\ p_\kappa^{(n+1)} \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{4} & 0 \\ \frac{1}{6} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & 0 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{3}{2} & 0 \end{pmatrix} \begin{pmatrix} (1 - p_t^{(n)}) \theta[1 - p_t^{(n)}] \\ (1 - p_b^{(n)}) \theta[1 - p_b^{(n)}] \\ (1 - p_\tau^{(n)}) \theta[1 - p_\tau^{(n)}] \\ (1 - p_\lambda^{(n)}) \Delta_{34}[q, 1 - p_\lambda^{(n)}] \\ (1 - p_\kappa^{(n)}) \Delta_{24}[q, 1 - p_\kappa^{(n)}] \end{pmatrix} \quad (\text{C.4})$$

where we define

$$\Delta_{ij}[q, x] \equiv (\delta_{qi} + \delta_{qj}) \theta[x] \quad (\text{C.5})$$

the δ 's are Kronecker's and θ is the Heaviside function. Equation (C.4) describes compactly the four IRQFP regimes labeled by $q = 1, 2, 3, 4$. The θ function is here to account in general for the fact that some of the p_i 's can become larger than one at some iteration order. It is important to keep this point under control since in a regime where some Y_i^0 is

	$Y_\kappa^0, Y_\lambda^0 < \infty$ (1)	$Y_\lambda^0 < \infty$ (2)	$Y_\kappa^0 < \infty$ (3)	/ (4)
p_t	7/61	7/61	4/31	7/61
p_b	19/61	19/61	10/31	19/61
p_τ	21/61	21/61	11/31	21/61
p_λ	58/61	296/183	29/31	296/183
p_κ	0	0	3/31	0

Table 1: Powers entering Eq. (C.2) in the various IRQFP regimes

very large, one has⁸ $1 + Y_i^0 \int_0^t u_i^{(n)} \sim Y_i^0 \int_0^t u_i^{(n)}$ if $p_i^{(n)} < 1$ and $1 + Y_i^0 \int_0^t u_i^{(n)} \sim 1$ if $p_i^{(n)} > 1$ (the critical value $p_i^{(n)} = 1$ is never met). Thus the proof of convergence to a unique form for the u_i^∞ 's (i.e. definite limiting values for the p_i 's and for the functions u_i^{FP}) needs some care. Direct numerical inspection of the iterations in (C.4) shows that only $p_\lambda^{(n)}$ can be either bigger or smaller than one. For regimes 1,2,3, $p_\lambda^{(n)}$ is found to be constantly smaller (or bigger) than one at each iteration. Regime 4 leads to some slight complications: $p_\lambda^{(n)}$ and $p_\kappa^{(n)}$ take alternating magnitudes, $p_\lambda^{(n)} > 1, p_\kappa^{(n)} > 0$ implying $p_\lambda^{(n+1)} < 1, p_\kappa^{(n+1)} = 0$ and vice versa up to the first six iterations, but then stabilize with $p_\lambda^{(n)} > 1, p_\kappa^{(n)} = 0$ for all $n \geq 7$. Armed with this numerical information, one can then consider Eq.(C.4) for any $n \geq 7$ to prove recursively that for all four regimes $0 < p_{t,b,\tau,\kappa}^{(n)} < 1$, while $0 < p_\lambda^{(n)} < 1$ for regimes 1,3 and $p_\lambda^{(n)} > 1$ for regimes 2, 4. Eliminating the θ functions correspondingly and solving Eq.(C.4) for $p_i^{(n+1)} = p_i^{(n)} = p_i$, we find the limiting values given in the Table 1.

The same result can be derived through the more systematic method of solving directly the linear system (C.4) for $p_i^{(n+1)} = p_i^{(n)} = p_i$, keeping formally the θ functions and not relying on any prior numerical information about the iterations. Then, checking the consistency of the solutions for all possible combinations of the p_i 's being greater or smaller than 1, one delineates the acceptable ones. It turns out that there is only one consistent solution for each regime leading to the values given in the table.

Knowing the p_i 's one can now extract the u_i^{FP} 's,

$$u_t^{\text{FP}} = \begin{cases} \frac{E_t}{(6r_b)^{\frac{1}{6}} (\int u_b^{\text{FP}})^{\frac{1}{6}}} & \text{regimes 1, 2 and 4} \\ \frac{E_t}{(6r_b)^{\frac{1}{6}} (4r_\lambda)^{\frac{1}{4}} (\int u_b^{\text{FP}})^{\frac{1}{6}} (\int u_\lambda^{\text{FP}})^{\frac{1}{4}}} & \text{regime 3} \end{cases} \quad (\text{C.6})$$

$$u_b^{\text{FP}} = \begin{cases} \frac{E_b}{(6)^{\frac{1}{6}} (4r_\tau)^{\frac{1}{4}} (\int u_t^{\text{FP}})^{\frac{1}{6}} (\int u_\tau^{\text{FP}})^{\frac{1}{4}}} & \text{regimes 1, 2 and 4} \\ \frac{E_t}{(6)^{\frac{1}{6}} (4r_\tau)^{\frac{1}{4}} (4r_\lambda)^{\frac{1}{4}} (\int u_t^{\text{FP}})^{\frac{1}{6}} (\int u_\tau^{\text{FP}})^{\frac{1}{4}} (\int u_\lambda^{\text{FP}})^{\frac{1}{4}}} & \text{regime 3} \end{cases} \quad (\text{C.7})$$

⁸provided that t is also large enough.

$$u_\tau^{\text{FP}} = \begin{cases} \frac{E_\tau}{(6r_b)^{\frac{1}{2}} (\int u_b^{\text{FP}})^{\frac{1}{2}}} & \text{regimes 1, 2 and 4} \\ \frac{E_t}{(6r_b)^{\frac{1}{2}} (4r_\lambda)^{\frac{1}{4}} (\int u_b^{\text{FP}})^{\frac{1}{2}} (\int u_\lambda^{\text{FP}})^{\frac{1}{4}}} & \text{regime 3} \end{cases} \quad (\text{C.8})$$

$$u_\lambda^{\text{FP}} = \begin{cases} \frac{E_\lambda}{(6)^{\frac{1}{2}} (6r_b)^{\frac{1}{2}} (4r_\tau)^{\frac{1}{4}} (\int u_t^{\text{FP}})^{\frac{1}{2}} (\int u_b^{\text{FP}})^{\frac{1}{2}} (\int u_\tau^{\text{FP}})^{\frac{1}{4}} (1+6Y_\kappa^0 t)^{\frac{2}{3}}} & \text{regime 1} \\ \frac{E_\lambda}{(6)^{\frac{1}{2}} (6r_b)^{\frac{1}{2}} (4r_\tau)^{\frac{1}{4}} (\int u_t^{\text{FP}})^{\frac{1}{2}} (\int u_b^{\text{FP}})^{\frac{1}{2}} (\int u_\tau^{\text{FP}})^{\frac{1}{4}}} & \text{regime 3} \\ \frac{E_\lambda}{(6)^{\frac{1}{2}} (6r_b)^{\frac{1}{2}} (4r_\tau)^{\frac{1}{4}} (6r_\kappa)^{\frac{2}{3}} (\int u_t^{\text{FP}})^{\frac{1}{2}} (\int u_b^{\text{FP}})^{\frac{1}{2}} (\int u_\tau^{\text{FP}})^{\frac{1}{4}} (\int u_\kappa^{\text{FP}})^{\frac{2}{3}}} & \text{regimes 2 and 4} \end{cases} \quad (\text{C.9})$$

$$u_\kappa^{\text{FP}} = \begin{cases} 1 & \text{regimes 1, 2 and 4} \\ \frac{E_\kappa}{(4r_\lambda)^{\frac{3}{2}} (\int u_\lambda^{\text{FP}})^{\frac{3}{2}}} & \text{regime 3} \end{cases} \quad (\text{C.10})$$

Finally let us note that p_κ is non vanishing (but small) only in regime 3. Consequently, the evolution of Y_κ will be very slow in this case, and a IRQFP regime will be theoretically obtained for very large values of Y_κ^0 (or very large values of r_κ), as it has been seen in the numerical analysis of section 4.

Appendix D: The soft parameters in the IRQFP regimes

In this appendix we discuss in some detail the sensitivity of the soft couplings and masses to their initial conditions in the IRQFP regimes. Keeping in mind that one can derive the results directly from the Yukawa couplings discussed in the previous appendix through the substitutions Eq.(3.10), it is instructive to make a direct study starting off directly from the analytical forms in the soft sector.

The A 's

To understand the behaviour of the soft trilinear couplings in the various IRQFP regimes let us study first that of the auxiliary functions given in (B.1) – (B.5). To start with, we take for illustration the case of e_t . Denoting by e_i^∞ 's the limit of the e_i 's when $Y_{t,b,\tau}^0$ tend to infinity, and assuming that t_0 is sufficiently large so that $1 + 6Y_b^0 \int_0^{t_0} u_b$ is well approximated by $6Y_b^0 \int_0^{t_0} u_b^\infty$, Equation (B.1) reads

$$e_t^\infty(t_0) = \frac{1}{E_t} \frac{d\tilde{E}_t}{d\eta}(t_0) + \frac{1}{6} A_b^0 - \frac{\int_0^{t_0} u_b^\infty e_b^\infty}{6 \int_0^{t_0} u_b^\infty} + Y_\lambda^0 \frac{A_\lambda^0 \int_0^{t_0} u_\lambda^\infty - \int_0^{t_0} u_\lambda^\infty e_\lambda^\infty}{1 + 4Y_\lambda^0 \int_0^{t_0} u_\lambda^\infty} \quad (\text{D.1})$$

The above equation is valid for any of the four regimes considered in section 3.2. To write more specific forms for each regime one uses Eq.(3.17) together with the corresponding values of the powers given in the table of Appendix C. One then obtains

$$e_t^\infty = \frac{1}{E_t} \frac{d\tilde{E}_t}{d\eta} - \frac{\int u_b^\infty e_b^\infty}{6 \int u_b^\infty} + \frac{A_b^0}{6} + \begin{cases} 0 & \text{regimes 1, 2 and 4} \\ -\frac{\int u_\lambda^\infty e_\lambda^\infty}{4 \int u_\lambda^\infty} + \frac{A_\lambda^0}{4} & \text{regime 3} \end{cases} \quad (\text{D.2})$$

In deriving the above equation for regimes 1, 2 and 4 we have made the implicit assumption that the magnitude of e_λ^∞ remains under control so that $\int u_\lambda^\infty e_\lambda^\infty \rightarrow 0$ with growing Y^0 .

Equations (B.2) – (B.5) yield along the same lines the corresponding e^∞ in the various IRQFP regimes:

$$e_b^\infty = \frac{1}{E_b} \frac{d\tilde{E}_b}{d\eta} - \frac{\int u_t^\infty e_t^\infty}{6 \int u_t^\infty} - \frac{\int u_\tau^\infty e_\tau^\infty}{4 \int u_\tau^\infty} + \frac{A_t^0}{6} + \frac{A_\tau^0}{4} + \begin{cases} 0 & \text{regimes 1, 2 and 4} \\ -\frac{\int u_\lambda^\infty e_\lambda^\infty}{4 \int u_\lambda^\infty} + \frac{A_\lambda^0}{4} & \text{regime 3} \end{cases} \quad (\text{D.3})$$

$$e_\tau^\infty = \frac{1}{E_t} \frac{d\tilde{E}_t}{d\eta} - \frac{\int u_b^\infty e_b^\infty}{2 \int u_b^\infty} + \frac{A_b^0}{2} + \begin{cases} 0 & \text{regimes 1, 2 and 4} \\ -\frac{\int u_\lambda^\infty e_\lambda^\infty}{4 \int u_\lambda^\infty} + \frac{A_\lambda^0}{4} & \text{regime 3} \end{cases} \quad (\text{D.4})$$

$$e_\lambda^\infty = \frac{1}{E_\lambda} \frac{d\tilde{E}_\lambda}{d\eta} - \frac{\int u_t^\infty e_t^\infty}{2 \int u_t^\infty} - \frac{\int u_b^\infty e_b^\infty}{2 \int u_b^\infty} - \frac{\int u_\tau^\infty e_\tau^\infty}{4 \int u_\tau^\infty} + \frac{A_t^0}{2} + \frac{A_b^0}{2} + \frac{A_\tau^0}{4} + \begin{cases} \frac{2Y_\kappa^0 A_\kappa^0 t - 2Y_\kappa^0 \int e_\kappa^\infty}{1 + 6Y_\kappa^0 t} & \text{regimes 1, 2 and 4} \\ 0 & \text{regime 3} \end{cases} \quad (\text{D.5})$$

$$e_\kappa^\infty = \frac{1}{E_\kappa} \frac{d\tilde{E}_\kappa}{d\eta} + \begin{cases} 0 & \text{regimes 1, 2 and 4} \\ -\frac{3 \int u_\lambda^\infty e_\lambda^\infty}{2 \int u_\lambda^\infty} + \frac{3A_\lambda^0}{2} & \text{regime 3} \end{cases} \quad (\text{D.6})$$

where we used Eq.(C.10) when convenient. At this level we should stress that the particular structure of the above equations will actually allow to solve *exactly* for the dependence of the e^∞ 's on the initial conditions A_i^0 . Indeed, on one hand the A_i^0 's enter linearly the inhomogeneous parts of the integral system of equations (D.2) – (D.6) with t independent coefficients. On the other hand, all the integrated parts which induce iteratively a dependence on the A 's are of the form $\frac{\int u_i^\infty e_i^\infty}{\int u_i^\infty}$ so that any substitution therein of the form $e_i^\infty \rightarrow \sum_j c_i^j A_j^0$, where the c_i^j 's are scale independent constants, will yield again the same kind of dependence. It is crucial that all the coefficients multiplying the A 's remain scale

α_i^j	A_t^0	A_b^0	A_τ^0	A_λ^0	A_κ^0
e_t^∞	$-\frac{2}{61} (-\frac{5}{31})$	$\frac{12}{61} (\frac{3}{31})$	$-\frac{3}{61} (-\frac{3}{31})$	$0 (\frac{9}{31})$	$0 (0)$
e_b^∞	$\frac{12}{61} (\frac{3}{31})$	$-\frac{11}{61} (-\frac{8}{31})$	$\frac{18}{61} (\frac{8}{31})$	$0 (\frac{7}{31})$	$0 (0)$
e_τ^∞	$-\frac{6}{61} (-\frac{6}{31})$	$\frac{36}{61} (\frac{16}{31})$	$-\frac{9}{61} (-\frac{17}{93})$	$0 (\frac{20}{93})$	$0 (0)$
e_λ^∞	$\frac{27}{61} (\frac{18}{31})$	$\frac{21}{61} (\frac{14}{31})$	$\frac{10}{61} (\frac{20}{93})$	$0 (-\frac{29}{93})$	$\frac{2Y_\kappa^0 t}{1+6Y_\kappa^0 t} (0)$
e_κ^∞	$0 (-\frac{27}{31})$	$0 (-\frac{21}{31})$	$0 (-\frac{10}{31})$	$0 (\frac{61}{31})$	$0 (1)$

Table 2: Resummed linear weights α_i^j of the A^0 's in the various e_i^∞ 's, i.e $\sum_j \alpha_i^j A_j^0 \subset e_i^\infty$. The unbracketed numbers are common to regimes 1,2,4 and the numbers between brackets correspond to regime 3.

independent at any order of iteration of the integral system of equations. It follows that one can re-sum the ensuing infinite series giving the numerical coefficients of the A^0 's. Equivalently, one can replace formally everywhere $\frac{\int u_i^\infty e_i^\infty}{\int u_i^\infty}$ by e_i^∞ and solve the resulting *linear* system in the e_i^∞ 's to obtain the exact linear dependence on the A 's. The e_i^∞ 's take then the following form

$$e_i^\infty = e_i^{\text{FP}} + \sum_j \alpha_i^j A_j^0 \quad (\text{D.7})$$

where the e_i^{FP} 's are completely independent of the A_j^0 's. The various coefficients α_i^j corresponding to the full resummation have been summarized in Table 2. Note that there are in Eq.(D.5), regimes 1, 2, 4, two terms not respecting the convenient structure. Nonetheless, due to a conspiracy among the various regimes, these terms do not invalidate the procedure described above for the determination of the coefficients α_i^j , even though α_λ^κ is scale dependent in regimes 1, 2, 4 (see Table 2).

As for the e_i^{FP} 's, their defining equations are the same as those for the e_i^∞ 's Eqs.(D.2) – (D.6) with all the A^0 's dropped out (the latter cancel automatically when Eq.(D.7) is used).

We have now all the ingredients to determine the dependence of the various A_i^∞ on their initial conditions. Since in all four regimes $Y_{t,b,\tau}^0$ are large $A_{i=t,b,\tau}^\infty$ take the form $-e_i^\infty + \frac{\int u_i^\infty e_i^\infty}{\int u_i^\infty} = -e_i^{\text{FP}} + \frac{\int u_i^{\text{FP}} e_i^{\text{FP}}}{\int u_i^{\text{FP}}}$, see Eqs.(3.12), (D.7), and are completely insensitive to the initial conditions A_i^0 similarly to the MSSM case [16]. In contrast A_λ, A_κ will be both sensitive to initial conditions in some regimes. Using Tables 1 and 2, equations (3.12), (C.2), (D.7) one finds in the regimes $\begin{cases} [1, 2, 4] \\ [3] \end{cases}$

$$A_\lambda^\infty = A_\lambda^{\text{FP}} - \left\{ \begin{array}{c} \frac{27}{61} \\ 0 \end{array} \right\} A_t^0 - \left\{ \begin{array}{c} \frac{21}{61} \\ 0 \end{array} \right\} A_b^0 - \left\{ \begin{array}{c} \frac{10}{61} \\ 0 \end{array} \right\} A_\tau^0 + \left\{ \begin{array}{c} 1 \\ 0 \end{array} \right\} A_\lambda^0 - \left\{ \begin{array}{c} \frac{2Y_\kappa^0 t}{1+6Y_\kappa^0 t} A_\kappa^0 [1] \\ \frac{1}{3} A_\kappa^0 [2, 4] \\ 0 [3] \end{array} \right\} \quad (\text{D.8})$$

and

$$A_\kappa^\infty = A_\kappa^{\text{FP}} + \left\{ \begin{array}{c} 0 \\ \frac{27}{31} \end{array} \right\} A_t^0 + \left\{ \begin{array}{c} 0 \\ \frac{21}{31} \end{array} \right\} A_b^0 + \left\{ \begin{array}{c} 0 \\ \frac{10}{31} \end{array} \right\} A_\tau^0 - \left\{ \begin{array}{c} 0 \\ \frac{61}{31} \end{array} \right\} A_\lambda^0 + \left\{ \begin{array}{c} \frac{1}{1+6Y_\kappa^0 t} A_\kappa^0 [1] \\ 0 [2, 4] \\ A_\kappa^0 [3] \end{array} \right\} \quad (\text{D.9})$$

where A_i^{FP} is defined in Eq.(3.21)

Hence, when one goes beyond the MSSM it is important to distinguish between e_i^{FP} and e_i^∞ , the former being useful intermediate functions to define the initial condition blind parts of the A_k in some IRQFP regimes.

Finally let us note that the sensitivity to the initial conditions in Eqs.(D.9), (D.8) does not imply that the physics is no more blind to these conditions. As was already stressed in section 3.2, the Yukawa couplings that multiply $A_{\lambda,\kappa}$ in the Lagrangian are vanishing in the corresponding IRQFP regimes so that the expected screening properties are always recovered at the level of the physical quantities.

The Σ 's

The study of the auxiliary functions ξ_i is technically more complicated than that of the e_i 's. The discussion goes along the same lines as in the previous section in that the scale independent initial conditions contributions can be easily resummed, but there also appear scale dependent contributions in the ξ_i 's which we should discuss carefully. To illustrate the case let us consider for simplicity the top/bottom sector switching off momentarily all other contributions. In this case Eq.(B.7) reads in the limit $Y_{t,b}^0 \rightarrow \infty$

$$\begin{aligned} \xi_t^\infty &= \frac{1}{E_t} \frac{d^2 \tilde{E}_t}{d\eta d\bar{\eta}} + \frac{2}{E_t} \frac{d\tilde{E}_t}{d\eta} \left[\frac{A_b^0}{6} - \frac{\int u_b^{\text{FP}} e_b^\infty}{6 \int u_b^{\text{FP}}} \right] + 7 \left[\frac{A_b^0}{6} - \frac{\int u_b^{\text{FP}} e_b^\infty}{6 \int u_b^{\text{FP}}} \right]^2 \\ &+ \frac{2A_b^0 \int u_b^{\text{FP}} e_b^\infty - \int u_b^{\text{FP}} \xi_b^\infty}{6 \int u_b^{\text{FP}}} - \frac{1}{6} (\Sigma_b^0 + (A_b^0)^2) \end{aligned} \quad (\text{D.10})$$

and similarly for Eq.(B.8),

$$\xi_b^\infty = \xi_t^\infty [t \leftrightarrow b] \quad (\text{D.11})$$

Apart from the constant terms $(\Sigma_{b,t}^0 + (A_{b,t}^0)^2)$ which can be fully resummed there is a non trivial dependence on the initial conditions in the above equations, either explicitly or implicitly through $e_{b,t}$, which would be difficult to handle in the iterated system (D.10),

(D.11). Fortunately the corresponding terms will actually all cancel out. To see this, it is convenient to make first the following change of variables in Eqs.(D.10), (D.11):

$$\xi_k^\infty = \tilde{\xi}_k^\infty + \rho_1^k \Sigma_k^0 + \rho_2^k (A_k^0)^2 + 2\rho_3^k A_k^0 e_k^\infty \quad (\text{D.12})$$

($k = t, b$), where ρ_1^k, ρ_2^k are arbitrary constants which can be taken equal to 1. In the ensuing equations for $\tilde{\xi}_t^\infty, \tilde{\xi}_b^\infty$ we use Eq.(D.7) to extract all the dependence on initial conditions in $e_{b,t}^\infty$. In the resulting equations we substitute for $\tilde{\xi}_b^\infty$ in $\tilde{\xi}_t^\infty$ and vice versa. If now ρ_3^k is chosen in Eq.(D.12) such that $\rho_3^{t,b} A_{t,b}^0 = \sum_j \alpha_{t,b}^j A_j^0$, see Eq.(D.7), and using Eqs.(D.2), (D.3) for the reduced top/bottom system, one finds that all terms that contain initial conditions and are scale dependent cancel out from the defining equations for $\tilde{\xi}_t^\infty$ and $\tilde{\xi}_b^\infty$ ⁹.

Thus we are lead to a situation to the one for e_t, e_b of the previous subsection. One can then similarly define

$$\tilde{\xi}_k^\infty = \xi_k^{\text{FP}} + \sum_j \gamma_k^j \Sigma_j^0 + \sum_{i,j} \gamma_k^{ij} A_i^0 A_j^0 \quad (\text{D.13})$$

and determine the γ 's by solving a linear system of equations. All the ingredients described here generalize to the complete $t, b, \tau, \lambda, \kappa$ system albeit tedious algebraic manipulations. We will not write in this appendix the rather complicated dependence of the ξ_i^∞ on the GUT scale values of the soft terms. We give here directly the dependence in the Σ_i 's. A useful remark is in order to understand this dependence. A direct calculation shows that the expression of $\Sigma_{i=t,b}^\infty$, Eq.(3.12) in the limit $Y_i^0 \rightarrow \infty$, is invariant under the substitutions $e_i^\infty \rightarrow \tilde{e}_i = e_i^\infty - \rho_3^i A_i^0, \xi_i^\infty \rightarrow \tilde{\xi}_i, \tilde{\xi}_i \rightarrow \xi_i^{\text{FP}}$ where $\tilde{\xi}_i, \xi_i^{\text{FP}}$ are as defined in Eqs.(D.12), (D.13). Equating further the free parameter ρ_3^i to $\sum_j \alpha_i^j A_j^0$ so that ξ_i^{FP} is guaranteed to be A^0, Σ^0 independent and $\tilde{e}_i \equiv e_i^{\text{FP}}$, all dependence on the initial conditions of the soft parameters drop out from Σ_i^∞ . Defining

$$\Sigma_i^{\text{FP}} = (A_i^{\text{FP}})^2 + 2e_i^{\text{FP}} A_i^{\text{FP}} + \xi_i^{\text{FP}} - \frac{\int u_i^{\text{FP}} \xi_i^{\text{FP}}}{\int u_i^{\text{FP}}} \quad (\text{D.14})$$

one has

$$\Sigma_{i=t,b,\tau}^\infty \equiv \Sigma_i^{\text{FP}} \quad (\text{D.15})$$

$$\Sigma_\lambda^\infty = \Sigma_\lambda^{\text{FP}} + \beta_\lambda^j \Sigma_j^0 \quad (\text{D.16})$$

and

$$\Sigma_\kappa^\infty = \Sigma_\kappa^{\text{FP}} + \beta_\kappa^j \Sigma_j^0 \quad (\text{D.17})$$

⁹The specific values of the $\alpha_{t,b}^j$ ($j = t, b$) are of course crucial for this cancellation. It should be clear however that in the simplified example of the top/bottom sector we consider, these values differ from those in table 2.

where the $\Sigma_{t,b,\tau}^{\text{FP}}$'s are initial conditions independent for all four IRQFP regimes while $\Sigma_{\lambda,\kappa}^\infty$'s can obviously still have some dependence in the regimes where Y_λ^0 or Y_κ^0 are not large. One finds for the latter in the regimes $\begin{Bmatrix} [1, 2, 4] \\ [3] \end{Bmatrix}$,

$$\Sigma_\lambda^\infty = \Sigma_\lambda^{\text{FP}} - \begin{Bmatrix} \frac{27}{61} & \\ 0 & \end{Bmatrix} \Sigma_t^0 - \begin{Bmatrix} \frac{21}{61} & \\ 0 & \end{Bmatrix} \Sigma_b^0 - \begin{Bmatrix} \frac{10}{61} & \\ 0 & \end{Bmatrix} \Sigma_\tau^0 + \begin{Bmatrix} 1 & \\ 0 & \end{Bmatrix} \Sigma_\lambda^0 - \begin{Bmatrix} d_\lambda^0 & [1] \\ \frac{1}{3}\Sigma_\kappa^0 & [2, 4] \\ 0 & [3] \end{Bmatrix} \quad (\text{D.18})$$

and

$$\Sigma_\kappa^\infty = \Sigma_\kappa^{\text{FP}} + \begin{Bmatrix} 0 & \\ \frac{27}{31} & \end{Bmatrix} \Sigma_t^0 + \begin{Bmatrix} 0 & \\ \frac{21}{31} & \end{Bmatrix} \Sigma_b^0 + \begin{Bmatrix} 0 & \\ \frac{10}{31} & \end{Bmatrix} \Sigma_\tau^0 - \begin{Bmatrix} 0 & \\ \frac{61}{31} & \end{Bmatrix} \Sigma_\lambda^0 + \begin{Bmatrix} d_\kappa^0 & [1] \\ 0 & [2, 4] \\ \Sigma_\kappa^0 & [3] \end{Bmatrix} \quad (\text{D.19})$$

with

$$d_\lambda^0 = \frac{2Y_\kappa^0 t}{1 + 6Y_\kappa^0 t} \left[\Sigma_\kappa^0 - \frac{(A_\kappa^0)^2}{1 + 6Y_\kappa^0 t} \right] \quad (\text{D.20})$$

and

$$d_\kappa^0 = \frac{(A_\kappa^0)^2}{(1 + 6Y_\kappa^0 t)^2} + \frac{\Sigma_\kappa^0 - (A_\kappa^0)^2}{1 + 6Y_\kappa^0 t} \quad (\text{D.21})$$

The ξ^{FP} read

$$\xi_t^{\text{FP}} = \frac{1}{E_t} \frac{d^2 \tilde{E}_t}{d\eta d\tilde{\eta}} - \frac{1}{E_t} \frac{d\tilde{E}_t}{d\eta} \frac{\int u_b^{\text{FP}} e_b^{\text{FP}}}{3 \int u_b^{\text{FP}}} - \frac{\int u_b^{\text{FP}} \xi_b^{\text{FP}}}{6 \int u_b^{\text{FP}}} + \frac{7}{36} \left(\frac{\int u_b^{\text{FP}} e_b^{\text{FP}}}{\int u_b^{\text{FP}}} \right)^2 + d_t^\lambda \quad (\text{D.22})$$

$$\begin{aligned} \xi_b^{\text{FP}} &= \frac{1}{E_b} \frac{d^2 \tilde{E}_b}{d\eta d\tilde{\eta}} - \frac{1}{E_b} \frac{d\tilde{E}_b}{d\eta} \left(\frac{\int u_t^{\text{FP}} e_t^{\text{FP}}}{3 \int u_t^{\text{FP}}} + \frac{\int u_\tau^{\text{FP}} e_\tau^{\text{FP}}}{2 \int u_\tau^{\text{FP}}} \right) - \frac{\int u_t^{\text{FP}} \xi_t^{\text{FP}}}{6 \int u_t^{\text{FP}}} - \frac{\int u_\tau^{\text{FP}} \xi_\tau^{\text{FP}}}{4 \int u_\tau^{\text{FP}}} \\ &+ \frac{7}{36} \left(\frac{\int u_t^{\text{FP}} e_t^{\text{FP}}}{\int u_t^{\text{FP}}} \right)^2 + \frac{5}{16} \left(\frac{\int u_\tau^{\text{FP}} e_\tau^{\text{FP}}}{\int u_\tau^{\text{FP}}} \right)^2 + \frac{1}{12} \frac{\int u_t^{\text{FP}} e_t^{\text{FP}}}{\int u_t^{\text{FP}}} \frac{\int u_\tau^{\text{FP}} e_\tau^{\text{FP}}}{\int u_\tau^{\text{FP}}} + d_b^\lambda \end{aligned} \quad (\text{D.23})$$

$$\xi_\tau^{\text{FP}} = \frac{1}{E_\tau} \frac{d^2 \tilde{E}_\tau}{d\eta d\tilde{\eta}} - \frac{1}{E_\tau} \frac{d\tilde{E}_\tau}{d\eta} \frac{\int u_b^{\text{FP}} e_b^{\text{FP}}}{\int u_b^{\text{FP}}} - \frac{\int u_b^{\text{FP}} \xi_b^{\text{FP}}}{2 \int u_b^{\text{FP}}} + \frac{3}{4} \left(\frac{\int u_b^{\text{FP}} e_b^{\text{FP}}}{\int u_b^{\text{FP}}} \right)^2 + d_\tau^\lambda \quad (\text{D.24})$$

$$\begin{aligned} \xi_\lambda^{\text{FP}} &= \frac{1}{E_\lambda} \frac{d^2 \tilde{E}_\lambda}{d\eta d\tilde{\eta}} - \frac{1}{E_\lambda} \frac{d\tilde{E}_\lambda}{d\eta} \left(\frac{\int u_t^{\text{FP}} e_t^{\text{FP}}}{\int u_t^{\text{FP}}} + \frac{\int u_b^{\text{FP}} e_b^{\text{FP}}}{\int u_b^{\text{FP}}} + \frac{\int u_\tau^{\text{FP}} e_\tau^{\text{FP}}}{2 \int u_\tau^{\text{FP}}} \right) \\ &- \frac{\int u_t^{\text{FP}} \xi_t^{\text{FP}}}{2 \int u_t^{\text{FP}}} - \frac{\int u_b^{\text{FP}} \xi_b^{\text{FP}}}{2 \int u_b^{\text{FP}}} - \frac{\int u_\tau^{\text{FP}} \xi_\tau^{\text{FP}}}{4 \int u_\tau^{\text{FP}}} \\ &+ \frac{3}{4} \left(\frac{\int u_t^{\text{FP}} e_t^{\text{FP}}}{\int u_t^{\text{FP}}} \right)^2 + \frac{3}{4} \left(\frac{\int u_b^{\text{FP}} e_b^{\text{FP}}}{\int u_b^{\text{FP}}} \right)^2 + \frac{5}{16} \left(\frac{\int u_\tau^{\text{FP}} e_\tau^{\text{FP}}}{\int u_\tau^{\text{FP}}} \right)^2 \\ &+ \frac{1}{2} \frac{\int u_t^{\text{FP}} e_t^{\text{FP}}}{\int u_t^{\text{FP}}} \frac{\int u_b^{\text{FP}} e_b^{\text{FP}}}{\int u_b^{\text{FP}}} + \frac{1}{4} \frac{\int u_t^{\text{FP}} e_t^{\text{FP}}}{\int u_t^{\text{FP}}} \frac{\int u_\tau^{\text{FP}} e_\tau^{\text{FP}}}{\int u_\tau^{\text{FP}}} + \frac{1}{4} \frac{\int u_b^{\text{FP}} e_b^{\text{FP}}}{\int u_b^{\text{FP}}} \frac{\int u_\tau^{\text{FP}} e_\tau^{\text{FP}}}{\int u_\tau^{\text{FP}}} + d_\lambda^\kappa \end{aligned} \quad (\text{D.25})$$

$$\xi_\kappa^{\text{FP}} = \frac{1}{E_\kappa} \frac{d^2 \tilde{E}_\kappa}{d\eta d\bar{\eta}} + d_\kappa^\lambda \quad (\text{D.26})$$

It turns out that in regimes 1, 2, and 4 one has $d_i^\lambda = 0$

while in regime 3, we have

$$d_t^\lambda = -\frac{2}{E_t} \frac{d\tilde{E}_t}{d\eta} \frac{\int u_\lambda^{\text{FP}} e_\lambda^{\text{FP}}}{4 \int u_\lambda^{\text{FP}}} - \frac{\int u_\lambda^{\text{FP}} \xi_\lambda^{\text{FP}}}{4 \int u_\lambda^{\text{FP}}} + \frac{5}{16} \left(\frac{\int u_\lambda^{\text{FP}} e_\lambda^{\text{FP}}}{\int u_\lambda^{\text{FP}}} \right)^2 + \frac{1}{12} \frac{\int u_b^{\text{FP}} e_b^{\text{FP}}}{\int u_b^{\text{FP}}} \frac{\int u_\lambda^{\text{FP}} e_\lambda^{\text{FP}}}{\int u_\lambda^{\text{FP}}} \quad (\text{D.27})$$

$$d_b^\lambda = -\frac{2}{E_b} \frac{d\tilde{E}_b}{d\eta} \frac{\int u_\lambda^{\text{FP}} e_\lambda^{\text{FP}}}{4 \int u_\lambda^{\text{FP}}} - \frac{\int u_\lambda^{\text{FP}} \xi_\lambda^{\text{FP}}}{4 \int u_\lambda^{\text{FP}}} + \frac{5}{16} \left(\frac{\int u_\lambda^{\text{FP}} e_\lambda^{\text{FP}}}{\int u_\lambda^{\text{FP}}} \right)^2 + \frac{\int u_\lambda^{\text{FP}} e_\lambda^{\text{FP}}}{\int u_\lambda^{\text{FP}}} \left(\frac{1}{12} \frac{\int u_t^{\text{FP}} e_t^{\text{FP}}}{\int u_t^{\text{FP}}} + \frac{1}{8} \frac{\int u_\tau^{\text{FP}} e_\tau^{\text{FP}}}{\int u_\tau^{\text{FP}}} \right) \quad (\text{D.28})$$

$$d_\tau^\lambda = -\frac{2}{E_\tau} \frac{d\tilde{E}_\tau}{d\eta} \frac{\int u_\lambda^{\text{FP}} e_\lambda^{\text{FP}}}{4 \int u_\lambda^{\text{FP}}} - \frac{\int u_\lambda^{\text{FP}} \xi_\lambda^{\text{FP}}}{4 \int u_\lambda^{\text{FP}}} + \frac{5}{16} \left(\frac{\int u_\lambda^{\text{FP}} e_\lambda^{\text{FP}}}{\int u_\lambda^{\text{FP}}} \right)^2 + \frac{1}{4} \frac{\int u_b^{\text{FP}} e_b^{\text{FP}}}{\int u_b^{\text{FP}}} \frac{\int u_\lambda^{\text{FP}} e_\lambda^{\text{FP}}}{\int u_\lambda^{\text{FP}}} \quad (\text{D.29})$$

For d_λ^κ we find

- regime 1

$$d_\lambda^\kappa = -\frac{4}{E_\kappa} \frac{d\tilde{E}_\kappa}{d\eta} Y_\kappa^0 \frac{\int e_\kappa^{\text{FP}}}{1+6Y_\kappa^0 t} - 2Y_\kappa^0 \frac{\int \xi_\kappa^{\text{FP}}}{1+6Y_\kappa^0 t} + 16(Y_\kappa^0)^2 \left(\frac{\int e_\kappa^{\text{FP}}}{1+6Y_\kappa^0 t} \right)^2 + Y_\kappa^0 \frac{\int e_\kappa^{\text{FP}}}{1+6Y_\kappa^0 t} \left(2 \frac{\int u_t^{\text{FP}} e_t^{\text{FP}}}{\int u_t^{\text{FP}}} + 2 \frac{\int u_b^{\text{FP}} e_b^{\text{FP}}}{\int u_b^{\text{FP}}} + \frac{\int u_\tau^{\text{FP}} e_\tau^{\text{FP}}}{\int u_\tau^{\text{FP}}} \right) \quad (\text{D.30})$$

- regimes 2 and 4

$$d_\lambda^\kappa = -\frac{2}{E_\kappa} \frac{d\tilde{E}_\kappa}{d\eta} \frac{\int e_\kappa^{\text{FP}}}{3t} - \frac{\int \xi_\kappa^{\text{FP}}}{3t} + \frac{4}{9} \left(\frac{\int e_\kappa^{\text{FP}}}{t} \right)^2 + \frac{\int e_\kappa^{\text{FP}}}{6t} \left(2 \frac{\int u_t^{\text{FP}} e_t^{\text{FP}}}{\int u_t^{\text{FP}}} + 2 \frac{\int u_b^{\text{FP}} e_b^{\text{FP}}}{\int u_b^{\text{FP}}} + \frac{\int u_\tau^{\text{FP}} e_\tau^{\text{FP}}}{\int u_\tau^{\text{FP}}} \right) \quad (\text{D.31})$$

- regime 3

$$d_\lambda^\kappa = 0 \quad (\text{D.32})$$

and for d_κ^λ

$$d_\kappa^\lambda = \begin{cases} 0 & \text{regimes 1, 2, and 4} \\ -\frac{3}{E_\kappa} \frac{d\tilde{E}_\kappa}{d\eta} \frac{\int u_\lambda^{\text{FP}} e_\lambda^{\text{FP}}}{\int u_\lambda^{\text{FP}}} - \frac{3}{2} \frac{\int \xi_\lambda^{\text{FP}}}{\int u_\lambda^{\text{FP}}} + \frac{15}{4} \frac{\int u_\lambda^{\text{FP}} e_\lambda^{\text{FP}}}{\int u_\lambda^{\text{FP}}} & \text{regime 3} \end{cases} \quad (\text{D.33})$$

The m 's.

When we develop the expressions (B16-B23) in the different FP regimes and replace the Σ_t by their dependence on the soft masses m_i^2 , we find solutions of the form

$$m_i^2 = (m_i^2)_{m^0} + (m_i^2)_{M^0} + (m_i^2)_{Tr} \quad (D.34)$$

where $(m_i^2)_{Tr}$ is given by (3.30) and the $(m_i^2)_{m^0}$ are written below. But, there is no compelling exact analytical solutions for the $(m_i^2)_{M^0}$. For the regimes 1,2 and 4 :

$$\begin{aligned} (m_Q)_{m^0}^2 &= \frac{85}{122}m_{Q_3}^{0^2} - \frac{17}{122}m_{U_3}^{0^2} - \frac{10}{61}m_{D_3}^{0^2} + \frac{5}{122}m_L^{0^2} + \frac{5}{122}m_{E_3}^{0^2} - \frac{15}{122}m_1^{0^2} - \frac{17}{122}m_2^{0^2} \\ (m_T)_{m^0}^2 &= -\frac{17}{61}m_{Q_3}^{0^2} + \frac{40}{61}m_{U_3}^{0^2} + \frac{4}{61}m_{D_3}^{0^2} - \frac{1}{61}m_L^{0^2} - \frac{1}{61}m_{E_3}^{0^2} + \frac{3}{61}m_1^{0^2} - \frac{21}{61}m_2^{0^2} \\ (m_B)_{m^0}^2 &= -\frac{20}{61}m_{Q_3}^{0^2} + \frac{4}{61}m_{U_3}^{0^2} + \frac{37}{61}m_{D_3}^{0^2} + \frac{6}{61}m_L^{0^2} + \frac{6}{61}m_{E_3}^{0^2} - \frac{18}{61}m_1^{0^2} + \frac{4}{61}m_2^{0^2} \\ (m_L)_{m^0}^2 &= \frac{15}{122}m_{Q_3}^{0^2} - \frac{3}{122}m_{U_3}^{0^2} + \frac{9}{61}m_{D_3}^{0^2} + \frac{87}{122}m_L^{0^2} - \frac{35}{122}m_{E_3}^{0^2} - \frac{17}{122}m_1^{0^2} - \frac{3}{122}m_2^{0^2} \\ (m_E)_{m^0}^2 &= \frac{15}{61}m_{Q_3}^{0^2} - \frac{3}{61}m_{U_3}^{0^2} + \frac{18}{61}m_{D_3}^{0^2} - \frac{35}{61}m_L^{0^2} + \frac{26}{61}m_{E_3}^{0^2} - \frac{17}{61}m_1^{0^2} - \frac{3}{61}m_2^{0^2} \\ (m_1)_{m^0}^2 &= -\frac{45}{122}m_{Q_3}^{0^2} + \frac{9}{122}m_{U_3}^{0^2} - \frac{27}{61}m_{D_3}^{0^2} - \frac{17}{122}m_L^{0^2} - \frac{17}{122}m_{E_3}^{0^2} + \frac{51}{122}m_1^{0^2} + \frac{9}{122}m_2^{0^2} \\ (m_2)_{m^0}^2 &= -\frac{51}{122}m_{Q_3}^{0^2} - \frac{63}{122}m_{U_3}^{0^2} + \frac{6}{61}m_{D_3}^{0^2} - \frac{3}{122}m_L^{0^2} - \frac{3}{122}m_{E_3}^{0^2} + \frac{9}{122}m_1^{0^2} + \frac{59}{122}m_2^{0^2} \\ (m_S)_{m^0}^2 &= \begin{cases} \frac{m_S^{0^2} + 396Y_\kappa^0 m_S^{0^2} - 132Y_\kappa^0 (A_\kappa^0)^2}{(1+396Y_\kappa^0)^2} & \text{regime 1} \\ m_S^{0^2} & \text{regime 2, 4} \end{cases} \quad (D.35) \end{aligned}$$

In the regime 3. we obtained

$$\begin{aligned} (m_Q)_{m^0}^2 &= \frac{39}{62}m_{Q_3}^{0^2} - \frac{11}{62}m_{U_3}^{0^2} - \frac{6}{31}m_{D_3}^{0^2} + \frac{5}{186}m_L^{0^2} + \frac{5}{186}m_{E_3}^{0^2} - \frac{5}{62}m_1^{0^2} - \frac{17}{186}m_2^{0^2} + \frac{8}{93}m_S^{0^2} \\ (m_T)_{m^0}^2 &= -\frac{11}{31}m_{Q_3}^{0^2} + \frac{19}{31}m_{U_3}^{0^2} + \frac{1}{31}m_{D_3}^{0^2} - \frac{1}{31}m_L^{0^2} - \frac{1}{31}m_{E_3}^{0^2} + \frac{3}{31}m_1^{0^2} - \frac{9}{31}m_2^{0^2} + \frac{3}{31}m_S^{0^2} \\ (m_B)_{m^0}^2 &= -\frac{12}{31}m_{Q_3}^{0^2} + \frac{1}{31}m_{U_3}^{0^2} + \frac{18}{31}m_{D_3}^{0^2} + \frac{8}{93}m_L^{0^2} + \frac{8}{93}m_{E_3}^{0^2} - \frac{8}{31}m_1^{0^2} + \frac{10}{93}m_2^{0^2} + \frac{7}{93}m_S^{0^2} \\ (m_L)_{m^0}^2 &= \frac{5}{62}m_{Q_3}^{0^2} - \frac{3}{62}m_{U_3}^{0^2} + \frac{4}{31}m_{D_3}^{0^2} + \frac{131}{186}m_L^{0^2} - \frac{55}{186}m_{E_3}^{0^2} - \frac{7}{62}m_1^{0^2} + \frac{1}{186}m_2^{0^2} + \frac{5}{93}m_S^{0^2} \end{aligned}$$

$$\begin{aligned}
(m_E)_{m^0}^2 &= \frac{5}{31}m_{Q_3}^{0^2} - \frac{3}{31}m_{U_3}^{0^2} + \frac{8}{31}m_{D_3}^{0^2} - \frac{55}{93}m_L^{0^2} + \frac{38}{93}m_{E_3}^{0^2} - \frac{7}{31}m_1^{0^2} - \frac{1}{93}m_2^{0^2} + \frac{10}{93}m_S^{0^2} \\
(m_1)_{m^0}^2 &= -\frac{15}{62}m_{Q_3}^{0^2} + \frac{9}{62}m_{U_3}^{0^2} - \frac{12}{31}m_{D_3}^{0^2} - \frac{7}{62}m_L^{0^2} - \frac{7}{62}m_{E_3}^{0^2} + \frac{21}{62}m_1^{0^2} - \frac{1}{62}m_2^{0^2} - \frac{5}{31}m_S^{0^2} \\
(m_2)_{m^0}^2 &= -\frac{17}{62}m_{Q_3}^{0^2} - \frac{27}{62}m_{U_3}^{0^2} + \frac{5}{31}m_{D_3}^{0^2} + \frac{1}{186}m_L^{0^2} + \frac{1}{186}m_{E_3}^{0^2} - \frac{1}{62}m_1^{0^2} + \frac{71}{186}m_2^{0^2} - \frac{17}{93}m_S^{0^2} \\
(m_S)_{m^0}^2 &= \frac{16}{31}m_{Q_3}^{0^2} + \frac{9}{31}m_{U_3}^{0^2} + \frac{7}{31}m_{D_3}^{0^2} + \frac{10}{93}m_L^{0^2} + \frac{10}{93}m_{E_3}^{0^2} - \frac{10}{31}m_1^{0^2} - \frac{34}{93}m_2^{0^2} + \frac{32}{93}m_S^{0^2}
\end{aligned} \tag{D.36}$$

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